Numerical Simulation of Lightning Induced Voltage on Power Transmission Lines

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Abstract - One of the reasons that over-voltages exist in electric power system is that lightning hits on a power line. Over-voltages are induced on the conductors when the lightning strikes on the transmission line. The over-voltages and thus over-currents can disturb and even damage the system. An understanding of lightning induced voltage and current on the power transmission lines is critical both in industry and in academia, for the protection of power systems. In this paper time domain finite difference method and time domain integral equation (TDIE) formulae are proposed to calculate the lightning-induced voltage and current along the transmission lines. The response of transmission lines to transient, Gaussian-pulsed electric field is simulated with typical transmission line configuration. Possible line protection schemes to minimize surge voltage and current are discussed. The simulation is beneficial for the understanding of the physics behind the theory.

Keywords: Transmission line, Lightning induced voltage and current, FDTD, Time-domain integral equation method

INTRODUCTION

Power transmission line performance can be affected by disturbances created by lightning flashes. These disturbances can be classified as over-currents and over-voltages due to direct lightning strikes to the line, and induced by indirect electromagnetic fields in the vicinity. These large, direct or induced current and voltage can be as large as to destroy the power equipment, especially for the medium and lower voltage lines.

DIRECT LIGHTNING STROKE

A direct lightning stroke is simulated by an ideal current source injected into the transmission line. The current takes the form of two double exponential pulses

$$i(t) = I_0 \left(e^{-\alpha t} - e^{-\beta t} \right), \tag{1}$$

where t is time variable. Typical characteristic parameters [Ahmad, 1] are: half-value time $T(0.5) = 50\mu s$; the time to crest of the pulse, $T_{cr} = 1.2\mu s$. The peak current amplitude $i_0 = 10$ kA; and reciprocals of time constant are $\alpha = -1400 s^{-1}$ and $\beta = -4.85 \times 10^6 s^{-1}$.

The plot of the input current is plotted in Figure 1.

The Chowdhuri and Gross model and the Agrawal model are two coupling models that are most frequently used in the power-lightning literature for the calculation of lightning-induced over-voltages. The Agrawal model [Agrawal, 2] was derived from Maxwell's equations and transmission line assumption.

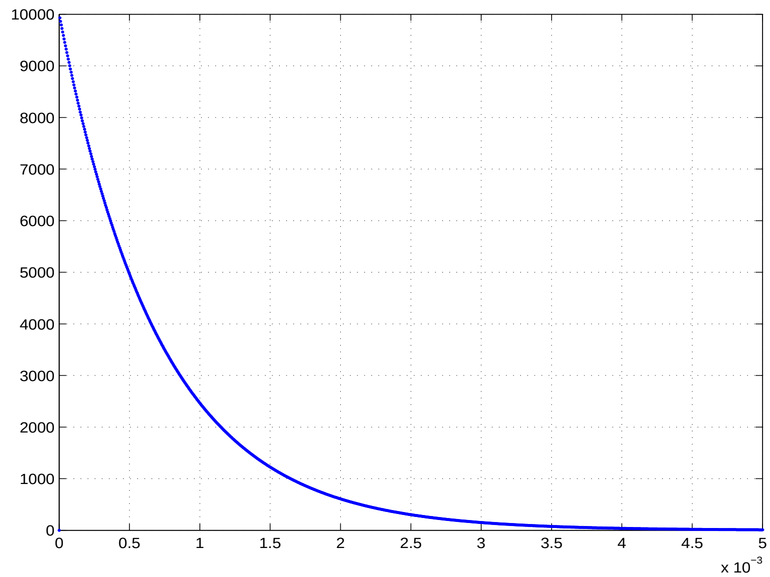

Figure 1: Incident double exponential current

$$\frac{\partial v^s(x,t)}{\partial t} + L\frac{\partial i(x,t)}{\partial t} = E_x^e(x,h,t) \tag{2}$$

$$\frac{\partial i(x,t)}{\partial t} + C\frac{\partial v^s(x,t)}{\partial t} = j(x,t) \tag{3}$$

where L and C are specific inductance and capacitance of the transmission line. $E_x^e(x,h,t)$ is the horizontal components of the incident electric field along the transmission line. $j(x,t)$ is the equivalent current source due to the vertical component of the electric field. $i(x,t)$ is the induced current along the line.

Finite difference time domain (FDTD) methods are usually used to solve the equations. Interested readers can find details on a scheme to discretize the above two equations in [Mimouni, 3].

INDIRECT LIGHTNING STROKE

In this section, we assume that the lightning does not strike the transmission line directly. Instead, the lightning generates an electric field nearby the line. Assuming the transmission line is thin and straight, the time domain Maxwell's equations in the differential form are

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}, \tag{4}$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} + \mathbf{J}^i(\mathbf{r},t), \tag{5}$$

$$\nabla \cdot \mathbf{D}(\mathbf{r},t) = q_v^i(\mathbf{r},t), \tag{6}$$

$$\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0. \tag{7}$$

$\mathbf{E}(\mathbf{r},t)$ is the electric field intensity; $\mathbf{H}(\mathbf{r},t)$is the magnetic field intensity; $\mathbf{D}(\mathbf{r},t)$ is the electric flux density; $\mathbf{B}(\mathbf{r},t)$ is the magnetic flux density; ε is the permittivity of the medium; μ is the permeability of the medium;

$\mathbf{J}^i(\mathbf{r},t)$ is the induced surface current density; $q_v^i(\mathbf{r},t)$is the induced volume charge density; $\mathbf{r}$ is the position vector and t is the time variable.

Assuming ε and μ are constants in the constitutive relations:

$$\mathbf{D}(\mathbf{r},t) = \varepsilon\mathbf{E}(\mathbf{r},t) \tag{8}$$

and

$$\mathbf{B}(\mathbf{r},t) = \mu\mathbf{H}(\mathbf{r},t), \tag{9}$$

we have

$$\nabla\times\mathbf{E}(\mathbf{r},t) = -\mu\frac{\partial\mathbf{H}(\mathbf{r},t)}{\partial t}, \tag{10}$$

$$\nabla\times\mathbf{H}(\mathbf{r},t) = \varepsilon\frac{\partial\mathbf{E}(\mathbf{r},t)}{\partial t} + \mathbf{J}^i(\mathbf{r},t), \tag{11}$$

$$\nabla\cdot\mathbf{E}(\mathbf{r},t) = \frac{q_v^i(\mathbf{r},t)}{\varepsilon}, \tag{12}$$

$$\nabla\cdot\mathbf{H}(\mathbf{r},t) = 0. \tag{13}$$

By taking the divergence of Equation 11, the continuity equation is obtained:

$$\nabla\cdot\mathbf{J}^i(\mathbf{r},t) = -\frac{\partial q_v^i(\mathbf{r},t)}{\partial t}. \tag{14}$$

By introducing the concept of vector and scalar potentials and defining

$$\mathbf{B}(\mathbf{r},t) \triangleq \nabla\times\mathbf{A}(\mathbf{r},t), \tag{15}$$

we have

$$\nabla\times\mathbf{E}(\mathbf{r},t) = -\nabla\times\frac{\partial\mathbf{A}(\mathbf{r},t)}{\partial t}, \tag{16}$$

or

$$\nabla\times\left(\mathbf{E}(\mathbf{r},t) + \frac{\partial\mathbf{A}(\mathbf{r},t)}{\partial t}\right) = 0. \tag{17}$$

Because the curl of a gradient is zero, it can be assumed

$$\mathbf{E}(\mathbf{r},t) + \frac{\partial\mathbf{A}(\mathbf{r},t)}{\partial t} = -\nabla\Phi(\mathbf{r},t), \tag{18}$$

where $\Phi(\mathbf{r},t)$ is a scalar potential defined later. $\mathbf{E}(\mathbf{r},t)$ is due to impressed and induced local charge sources. The scattered electric field $\mathbf{E}^s(\mathbf{r},t)$ due to induced charges is

$$\mathbf{E}^s(\mathbf{r},t) + \frac{\partial\mathbf{A}(\mathbf{r},t)}{\partial t} = -\nabla\Phi(\mathbf{r},t), \tag{19}$$

where vector potential $\mathbf{A}(\mathbf{r},t)$ is computed from the induced current by

$$\mathbf{A}(\mathbf{r},t) = \frac{\mu}{4\pi}\int_v \frac{\mathbf{J}^i(\mathbf{r}',t^r)}{R}dv', \tag{20}$$

and scalar potential $\Phi(\mathbf{r},t)$ is computed from the induced charges by

$$\Phi(\mathbf{r},t) = \frac{1}{4\pi\varepsilon}\int_{v}\frac{q_v^i(\mathbf{r}',t^r)}{R}dv'. \tag{21}$$

In the above equations, μ and ε denote the permeability and permittivity of the surrounding medium. $\mathbf{r}$ and $\mathbf{r}'$ are vectors from the origin to the observation and source points on the scatterer, respectively. Retarded time $t^r = t - t^d$ and t^d is the time for the wave to travel the distance between the observation and the source points. $\mathbf{J}^i(\mathbf{r}',t)$ and $q_v^i(\mathbf{r}',t)$ are the induced volume current density and volume charge density.

In the case of wire scatterers, we will replace $\mathbf{J}^i(\mathbf{r}',t)$ and $q_v^i(\mathbf{r}',t)$ with line current $\mathbf{I}(\mathbf{r}',t)$ and linear charge density $q_l(\mathbf{r}',t)$.

On the surface of the assumed, perfectly electric conducting (PEC) structure, the electromagnetic boundary condition requires that

$$\left[\mathbf{E}^i(\mathbf{r},t) + \mathbf{E}^s(\mathbf{r},t)\right]_t = 0, \tag{22}$$

where $\mathbf{E}^i(\mathbf{r},t)$ is the incident field and subscript $_t$ stands for tangential component. Substituting Equation 19 into Equation 22 produces

$$\left[\frac{\partial \mathbf{A}(\mathbf{r},t)}{\partial t} + \nabla\Phi(\mathbf{r},t)\right]_t = \left[\mathbf{E}^i(\mathbf{r},t\right]_t. \tag{23}$$

Equation 23 is the time domain electric field integral equation (TD EFIE) used to analyze general scattering problems.

In Equation 23, the vector potential $\mathbf{A}(\mathbf{r},t)$ and scalar potential $\Phi(\mathbf{r},t)$ are functions of induced current density and charge density, both of which need to be discretized and solved. For thin wire scatterers, the linear charge density $q_l(\mathbf{r},t)$ is related to the induced line current $I(\mathbf{r},t)$ by continuity equation

$$\frac{\partial q_l(\mathbf{r},t)}{\partial t} = -\frac{\partial I(\mathbf{r},t)}{\partial l}, \tag{24}$$

where l is the parameter along the length of the wire scatterer.

We can further simplify Equation 23 by time-differentiating it to obtain a TDIE equation for scattering analysis of arbitrary wire structure:

$$\left[\frac{\partial \mathbf{E}^i(\mathbf{r},t)}{\partial t}\right]_t = \left[\frac{\partial^2 \mathbf{A}(\mathbf{r},t)}{\partial t^2}\right]_t + \left[\nabla\frac{\partial\Phi(\mathbf{r},t)}{\partial t}\right]_t. \tag{25}$$

This way $\frac{\partial\Phi(\mathbf{r},t)}{\partial t}$ can be calculated from the induced line current instead of the induced linear charges:

$$\frac{\partial\Phi(\mathbf{r},t)}{\partial t} = -\frac{1}{4\pi\varepsilon}\int_{l'}\frac{\frac{\partial I(r,t^r)}{\partial l}}{R}dl'. \tag{26}$$

For simplicity, we denote

$$\Psi \triangleq \frac{\partial\Phi(\mathbf{r},t)}{\partial t} \tag{27}$$

in the following sections.

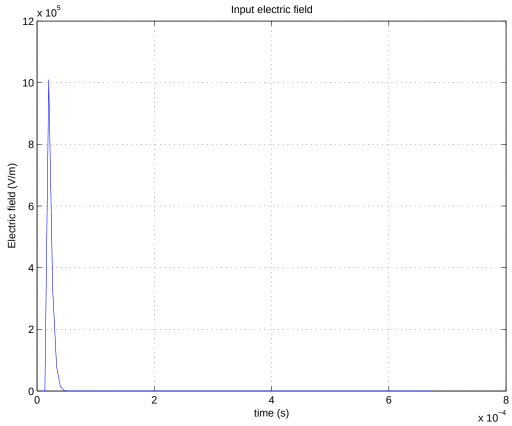

Figure 2: Incident Gaussian pulsed electric field

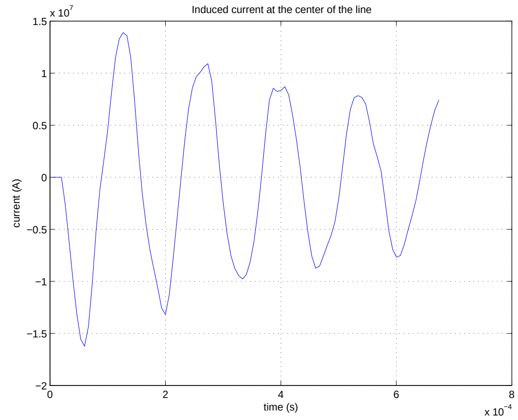

Figure 3: Induced current at the center of the line

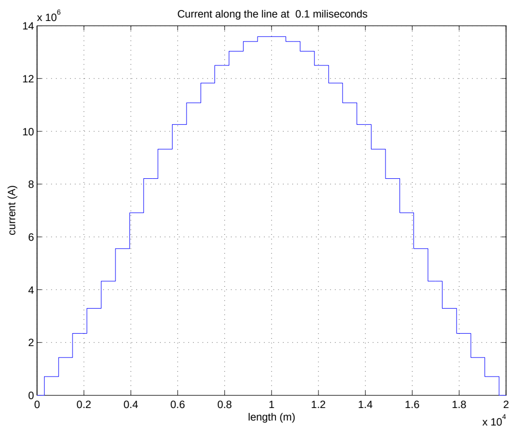

Figure 4: Induced current along the wire at one time

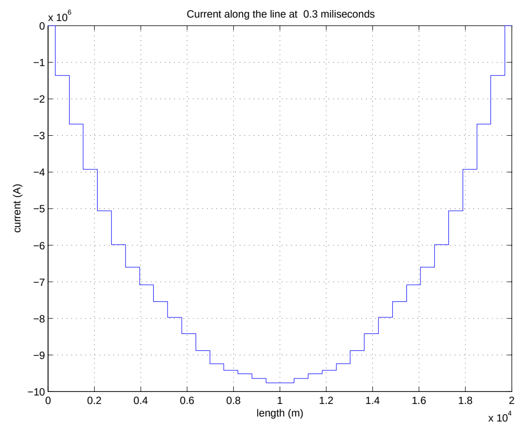

Figure 5: Induced current along the wire at another time

The Lorentz gauge condition is

$$\nabla \cdot \mathbf{A}(\mathbf{r},t) = -\frac{1}{c^2}\frac{\partial \Phi(\mathbf{r},t)}{\partial t}, \tag{28}$$

where c is the speed of light in the medium,

$$c = \frac{1}{\sqrt{\varepsilon\mu}}. \tag{29}$$

Applying it to Equation 25, we have an alternate expression of the TDIE:

$$\left[\frac{\partial \mathbf{E}^i(\mathbf{r},t)}{\partial t}\right]_t = \left[\frac{\partial^2 \mathbf{A}(\mathbf{r},t)}{\partial t^2}\right]_t - c^2\left[\nabla(\nabla\cdot\mathbf{A}(\mathbf{r},t))\right]_t. \tag{30}$$

Based on the measured data in [Rai, 4], the peak electrical field strength is taken as 10^6 V/m. From the measurement, we see that the radiation spectrum of lightning is represented well by a Gaussian pulse, as illustrated in Figure 2, instead of the double exponential form.

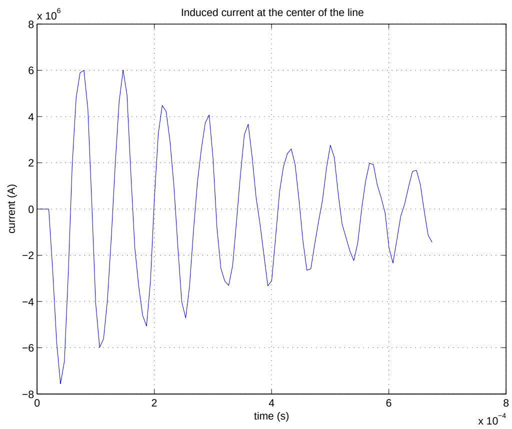

Figure 6: Induced current at the center of the line of 10 km

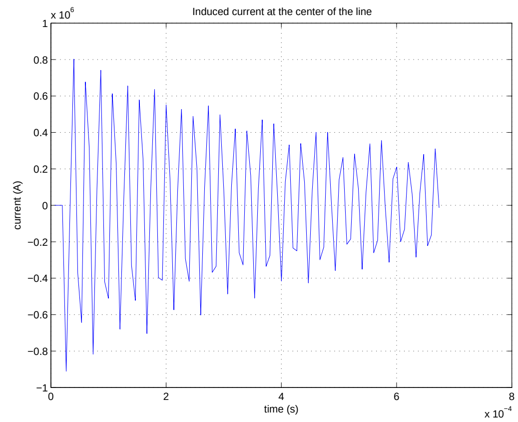

Figure 7: Induced current at the center of the line of 2 km

The line length is taken to be 20 kilometers; the peak electrical field strength is taken as 10^6 V/m at the line, although the actual value could be much smaller because of the perpendicular distance of the lightning stroke from the transmission line. The Gaussian distribution parameters are chosen so that the lighting spectra are centered at 15 KHz.

With the algorithms developed above, the induced current at the center of the transmission line with time is plotted in Figure 3; the currents along the line at two time instants are plotted in Figures 4 and 5. The stair-type plots in both figures are used to represent the effect of FDTD calculation.

The length of the transmission line also affects the induced currents. The induced currents with time along two transmission lines of lengths 10 and 2 kilometers are plotted in Figures 6 and 7. Since the transmission line is a resonant structure, the filtering effect plays an important role in the amplitude of the induced current. For the safety of the transmission line and its power equipment, protection devices should be installed at proper intervals based on the simulation results for best performance. From figures 3 and 4 we see that for electrically short lines, an absorbing device can be installed at the center of the line. For electrically long lines, these peaks of the standing wave can be determined by simulation.

Conclusions

Formulations from finite difference time domain method and time domain integral equation method are given to calculate the over-voltage and over-current due to direct lightning strike and induced by indirect electric fields. Based on the measurement results in literature, the electric filed is given in the form of time domain Gaussian pulse, having a Gaussian-pulsed spectrum. With simulation results of several examples, it is shown that the induced current is affected by the length of the transmission line.

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Zhaoxian Zhou received the B. Eng. from the University of Science and Technology of China in 1991; M. Eng. from the National University of Singapore in 1999 and the PhD degree from the University of New Mexico in 2005. All degrees are in Electrical Engineering. From 1991 to 1997, he was an Electrical Engineer in China Research Institute of Radiowave Propagation. In the fall of 2005, he joined the School of Computing, the University of Southern Mississippi as an assistant professor. His research interests include electromagnetics, radiowave propagation, high performance computing and numerical analysis. His teaching interests include communications, electromagnetics, antennas and propagation, electric power, and signal processing. He is a senior member of IEEE and a member of ASEE.