

# Using Interactive Virtual Instruments to Teach Spectral Analysis

*Y. Charles Lu<sup>1</sup>, Adrian Ieta<sup>2</sup>, William E. Murphy<sup>3</sup>*

**Abstract** – Spectral analysis, or Fourier analysis, is a very fundamental topic in experimentation courses in mechanical engineering programs. It deals with time-varying signals measured from various physical phenomena and involves concepts like waveforms, Fourier series, Fourier integrals, Fourier transform, sampling and aliasing, etc. The traditional method for teaching spectral analysis is mostly done by lecturing the mathematical developments of each concept with in-class quizzes, homework and separate laboratory exercises. While students may learn the mathematical procedures to compute simple waveforms, many do not fully understand how a complex spectral is generated and why a real-time, continuous signal may be distorted after data acquisition process. We have taken the interactive virtual instruments (LabView) into the classroom and used them to teach spectral analysis. These virtual instruments allow students to easily build and analyze various spectrums and instantaneously observe the outcomes. By using this integrated approach, the students are involved in the teaching process. As a result, the learning outcomes are accomplished.

*Keywords:* Virtual Instruments, Spectral analysis, LabView, Engineering experimentation.

## INTRODUCTION

Labview has been widely used in teaching laboratory practices in engineering and physics classes [1-8]. A graphical programming language, Labview allows the students to build virtual instruments (VI) on computer screens for data acquisition, signal analysis and instrument control. The University of Kentucky has adopted the Labview software for assisting the laboratory experiments and the feedback from the students has been very positive. We have also taken Labview into the classroom and used it to teach many fundamental aspects of mechanical measurements, such as spectral analysis.

Spectral analysis, also known as Fourier analysis, is a very fundamental topic in mechanical measurements. It deals with the time-varying signals used in the mechanical systems and the process of decomposing complex signals into basic functions from which the frequency response

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of the signals can be determined. Spectral analysis involves fundamental concepts like complex waveforms, Fourier series, Fourier integrals, Fourier transform, sampling and aliasing, etc. The Fourier analysis is a branch of classic mathematics that is often taught to senior or graduate students majored in mathematics. The traditional method of teaching Fourier analysis in mechanical measurement courses is by lecturing on the mathematical developments of Fourier analysis, have students complete homework assignments, and then assess the students with an exam. While students may learn the mathematical procedures to compute simple waveforms, many do not fully understand how a complex spectrum is produced or decomposed and why a real-time, continuous signal can be distorted after data acquisition process. Alternatively, Fourier analysis may be taught through laboratory exercises. That often requires the use of sophisticated instruments such as fast Fourier transform analyzers (FFT), function generators (multimeters), oscilloscopes, etc. Students may end up spending more time on learning the equipment rather than on the spectral analysis itself.

In the new approach, we integrated the interactive Labview simulations into the lectures to help students better understand the fundamental concepts in spectral analysis. We have moved the class from a traditional lecture room to an interactive television video (ITV) room that is fully equipped with computers. With the use of virtual instrument (VI) via Labview, students can not only learn the mathematical developments of various spectrums from the instructors but also have the opportunities to “build and play” with the spectrums. Since the students are highly engaged in the teaching process, the learning outcomes have been significantly improved.

The textbook used for this class (ME311-Engineering Experimentation II) is “Mechanical Measurements”, by Beckwith, Maragoni and Lienhard [9]. The spectral analysis is discussed primarily in Chapter 4 (The analogy Measurement: Time-Dependent Characteristics). The lecture is a two-way interaction: the instructors explain the mathematical development of each concept in spectral analysis and then the students build various spectrums via virtual instruments on their computers. Some of the examples used in the lectures are presented in this paper.

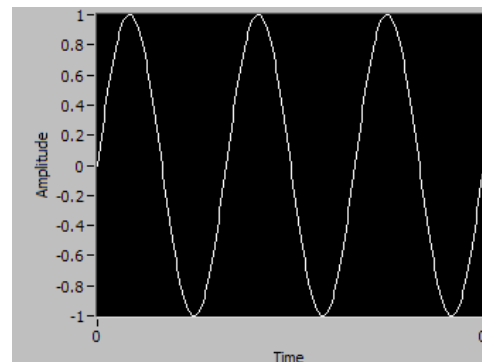
## SIMPLE HARMONIC AND COMPLEX FUNCTIONS

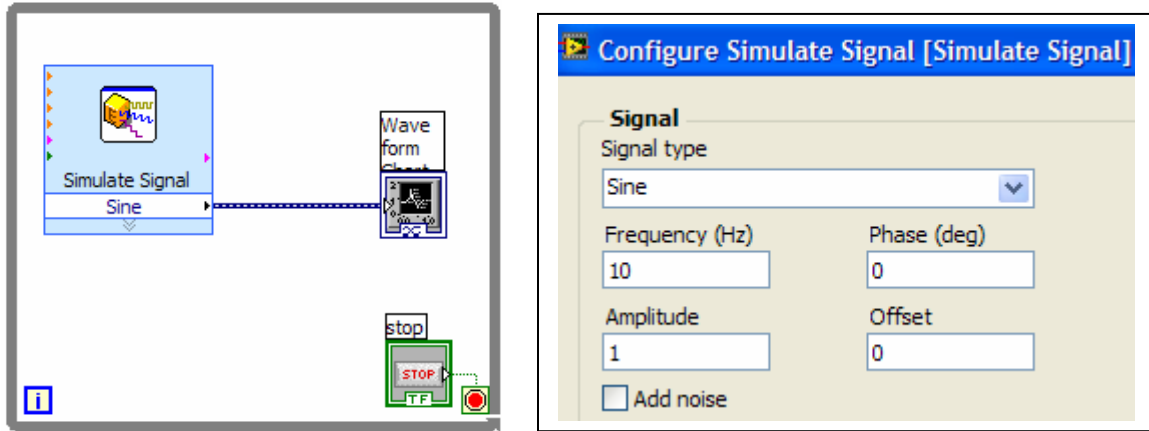
### Harmonic function

The chapter starts with the simple harmonic function, which is defined as the signal where its second derivative is proportional to the function but of opposite sign, such as a sine or cosine function [Equation 1]. The students can quickly build a very simple Labview Block Diagram (Figure 1b) and the resultant waveform is displayed on the Labview Front Panel (Figure 1a). Double click the Simulate Signal icon and a Configuration Window opens, which allows students to change the signal type and its characteristics (frequency, amplitude, phase and offset).

$$s = s_0 \sin(\omega t + \Phi) \quad [1]$$

where  $s_0$  - amplitude  
 $\omega$  - circular frequency,  $\omega=2\pi f$   
 $f$  - cyclic frequency (Hz)  
 $\Phi$  - phase angle

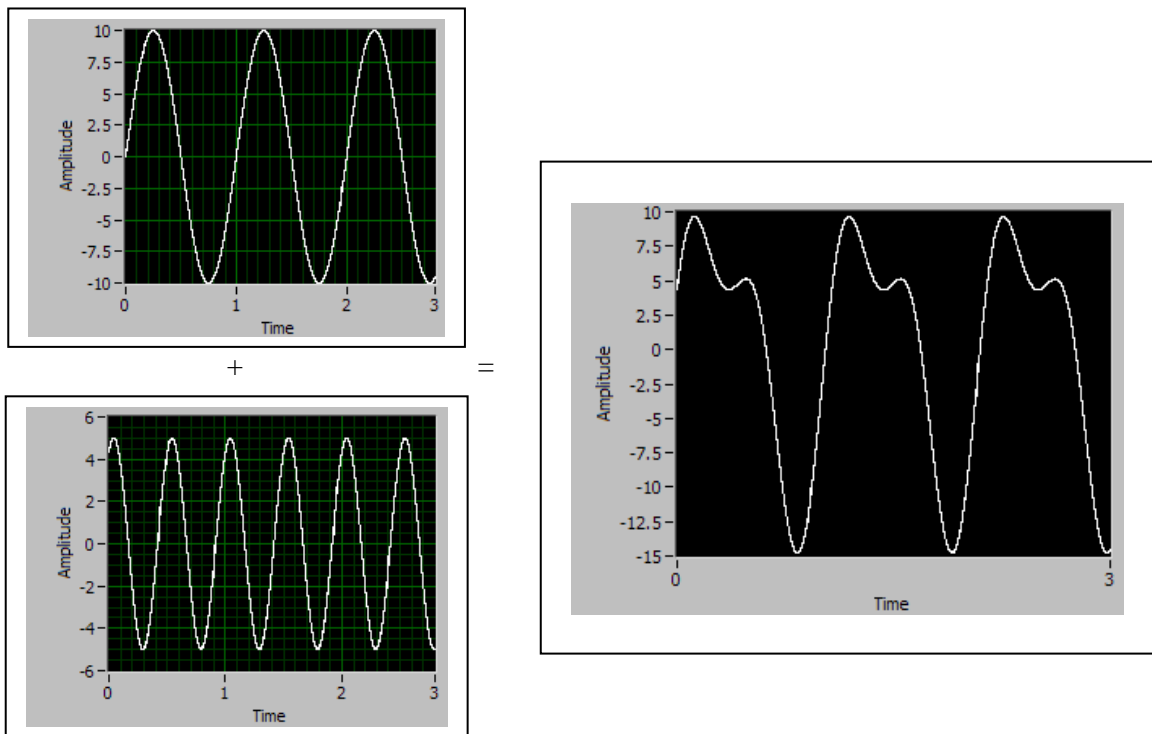




**Figure 1 – Labview Front Panel and Block Diagram for a simple harmonic signal.**

Complex functions

Actual mechanical signals such as sound, pressure, acceleration, etc. are often in complex forms, which are the combination of many simple harmonic functions that are in different amplitudes, frequencies and phases. For example, when two very simple sine functions  $f_{(1)}=10\sin\omega t$  and  $f_{(2)}=5\sin(2\omega t+\pi/3)$  are combined together, a complex signal is the result (Figure 2). To observe this, the students can quickly add another Simulate Signal to the previous Labview Block Diagram and then see the resultant complex waveform on the Front Panel. By changing the characteristics of the two sine functions, the students can observe how the resultant complex function evolves.



**Figure 2 – Labview Front Panel showing that two simple harmonic waveforms have produced a complex waveform.**

## Fourier Series

After discussing some relatively simple complex spectrums, we come to conclude that any dynamic signal may be considered as a summation of many sine and cosine signals (Figure 3). The mathematical formula of a generic complex spectrum is given by Fourier series (Equation 2):

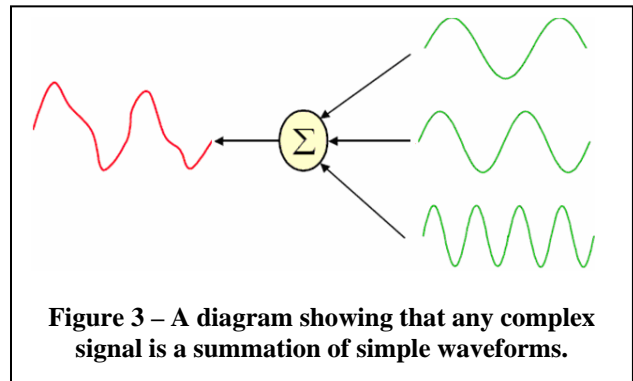
$$f(t) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi}{p} t + b_n \sin \frac{n\pi}{p} t \right) \quad [2]$$

where

$$a_0 = \frac{1}{2p} \int_{-p}^p f(t) dt \quad [2a]$$

$$a_n = \frac{1}{p} \int_{-p}^p f(t) \cos \frac{n\pi}{p} dt \quad [2b]$$

$$b_n = \frac{1}{p} \int_{-p}^p f(t) \sin \frac{n\pi}{p} dt \quad [2c]$$

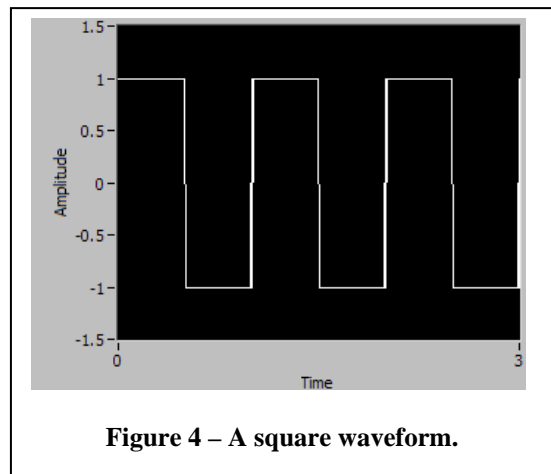


For example, the Fourier series of a square waveform (a common waveform used in engineering application such as the pressure history in a chemical reactor) may be obtained as follows

- Since the function is piecewise and continuous for all  $t$ , it can be expressed within  $[0, 2p]$  as

$$f(t) = \begin{cases} 1 & \text{if } 0 < t < p, \\ -1 & \text{if } p < t < 2p. \end{cases} \quad [3]$$

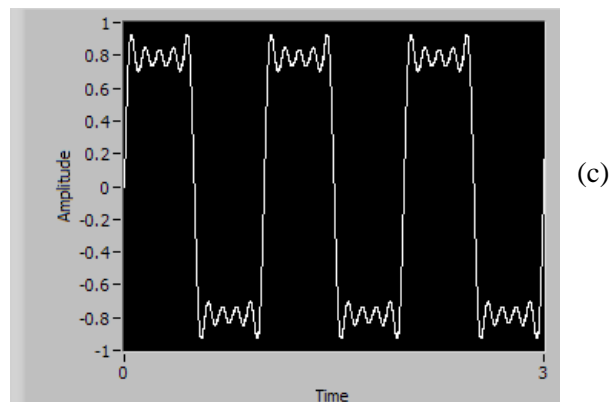
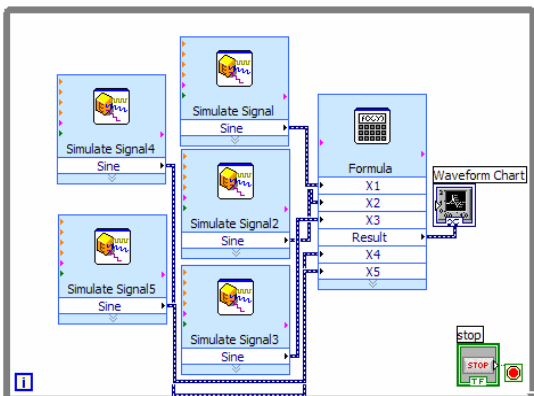
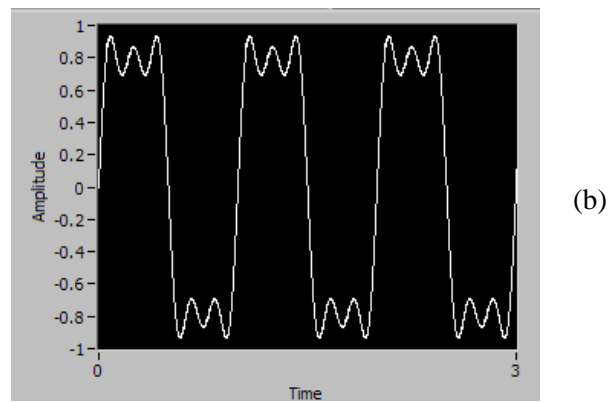
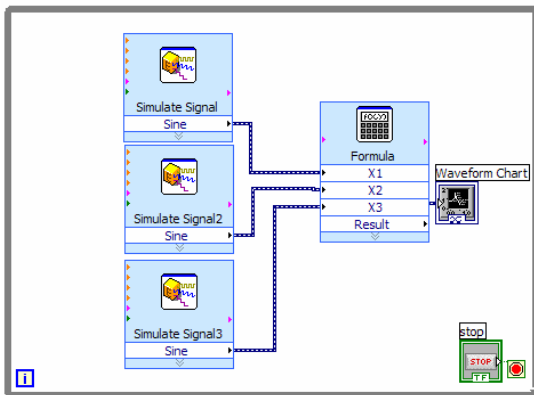
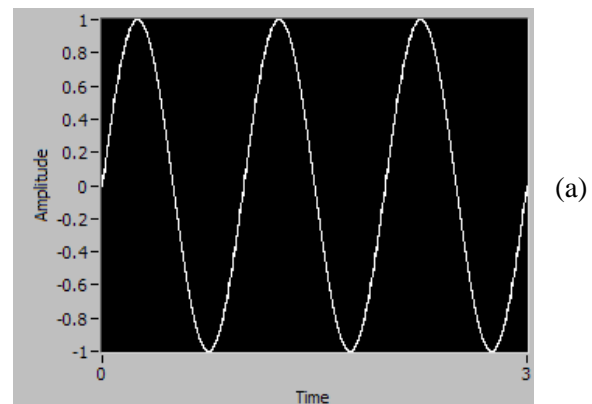
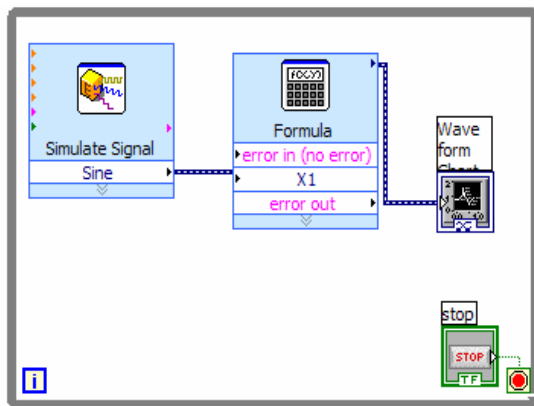
- Solving for  $a_0, a_n, b_n$ , via Equations 2a, 2b, 2c, the Fourier function of the square wave is obtained as

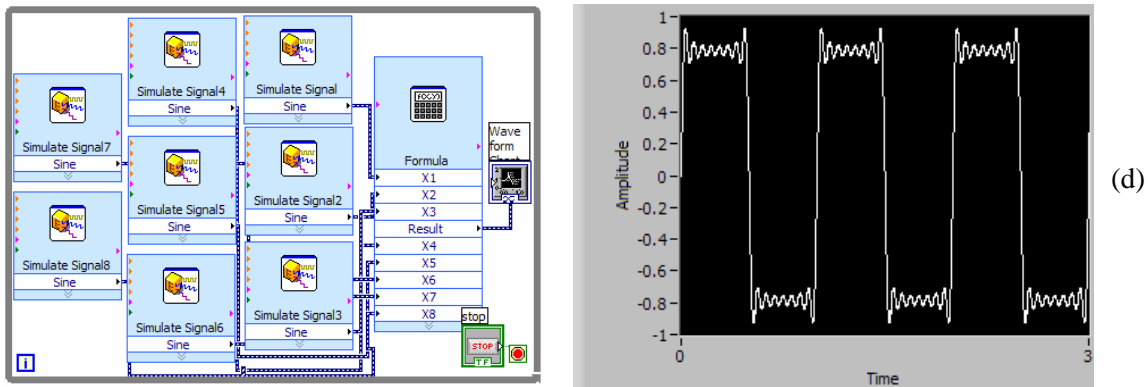


$$f(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n+1)\pi t}{(2n+1)p}$$

$$= \frac{4}{\pi} \left( \sin pt + \frac{1}{3} \sin 3pt + \frac{1}{5} \sin 5pt + \frac{1}{7} \sin 7pt + \frac{1}{9} \sin 9pt + \dots \right) \quad [4]$$

Now the students can add additional Simulate Signals to the Block Diagram they created earlier to construct the square waveform. Depending on the number of the sine functions used, the square waveform can be generated (Figures 5a-d).



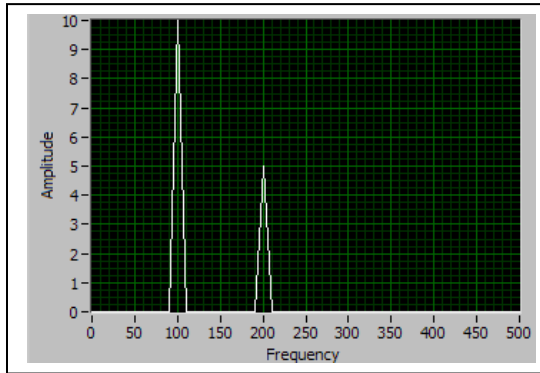
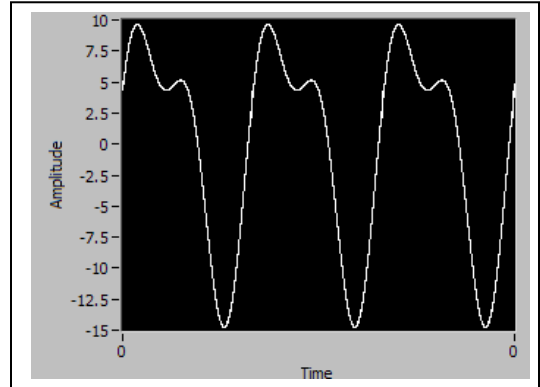
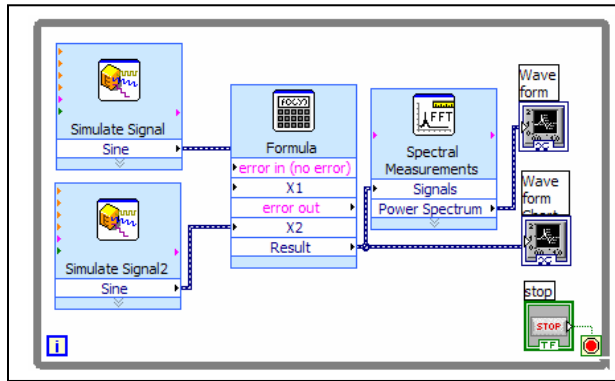


**Figure 5 – Labview Block Diagrams and Front Panels showing that a square waveform is produced via Fourier series. (a) Result of Fourier series with one sine term; (b) Result of Fourier series with three sine terms; (c) Result of Fourier series with five sine terms; and (d) Result of Fourier series with eight sine terms.**

### FOURIER TRANSFORM

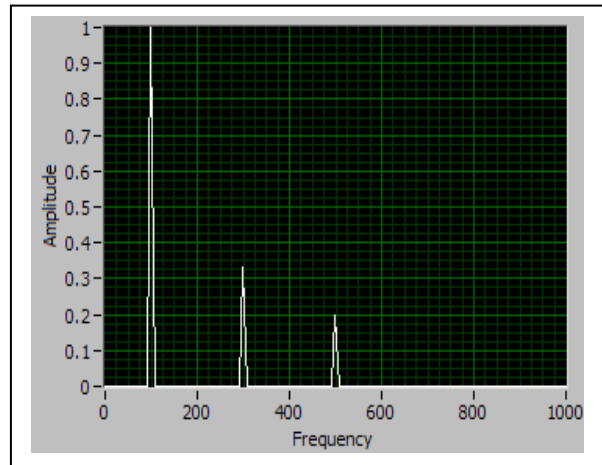
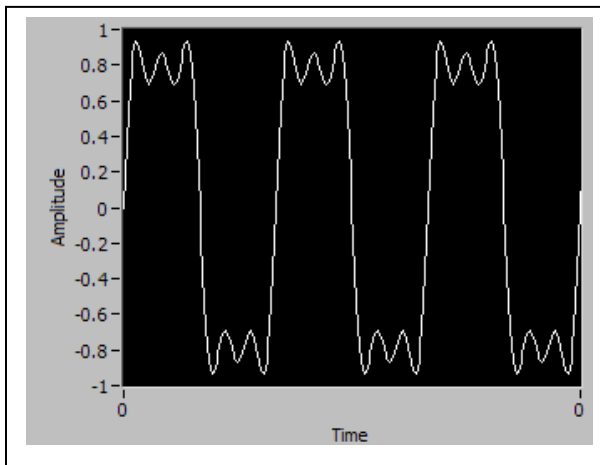
While the natural inclination of most people is to think of physical signals in time domain, many engineering applications requires the looking at the response of a system in frequency domain, or the frequency spectrum. For example, by looking at the frequency spectrum resulting from the vibration test of a structure, we could identify the structure’s natural frequencies which are critical for designing a safe and high quality product.

When a time-varying signal is transformed into frequency domain, the frequency spectrums of the signal can be identified. This process is called the Fourier transform. As an example, the frequency spectrum of the complex signal shown in Figure 2 can be easily constructed from its waveform equation:  $f(t)=10\sin\omega t +5\sin(2\omega t+\pi/3)$ . There are only two frequency spectrums, with frequencies of  $\omega$  and  $2\omega$ . This process can be viewed through Labview. By adding a Fast Fourier Transform, or FFT, to the Block Diagram, the frequency spectrum is displayed on the Front Panel (Figure 6).



**Figure 6 – Labview Block Diagram and Front Panel showing the Fourier transform process.**

Similarly, a square waveform with three sine functions can have a frequency spectrum as shown in Figure 7.



**Figure 7 – Labview Block Diagram and Front Panel showing the Fourier transform of a square waveform.**

## DISCRETE FOURIER ANALYSIS

One very important subject in spectral analysis is the discrete Fourier analysis. It deals with how to properly measure (or record) a complex signal coming from a physical system. When we use a computer to record a “continuous” signal, we actually use “discrete” points taken at a specified time interval ( $\Delta t$ ). The inverse of the time interval is called the sampling rate:  $f_s = 1/\Delta t$ . If we record a total of  $N$  point over the time, then the total time is  $T=N\Delta t$ , or the fundamental frequency is  $\Delta f = 1/T = 1/N\Delta t$ . In practical engineering applications, the discrete Fourier series, instead of the ordinary Fourier series (Equation 2), is used for recording spectrums coming form the sensors/transducers and for data analysis.

By replacing the continuous time  $t$  in Equation 2 with the discrete time  $t_r = r\Delta t$ , the discrete Fourier series can be expressed as follows

$$f(t_r) = a_0 + \sum_{n=1}^{N/2-1} \left[ a_n \cos \frac{2\pi r n}{N} + b_n \sin \frac{2\pi r n}{N} \right] + \frac{A_{N/2}}{2} \cos(\pi r) \quad [5]$$

where

$$a_n = \frac{2}{N} \sum_{r=1}^N y(r\Delta t) \cos\left(\frac{2\pi r n}{N}\right), \quad n=, 0, 1, \dots, N/2 \quad [5a]$$

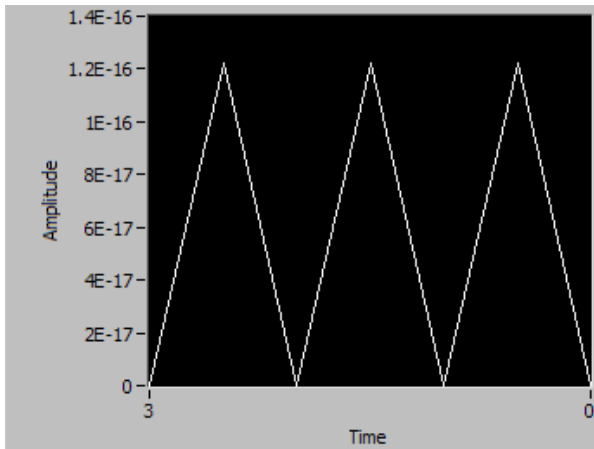
$$b_n = \frac{2}{N} \sum_{r=1}^N y(r\Delta t) \sin\left(\frac{2\pi r n}{N}\right), \quad n=, 0, 1, \dots, N/2 \quad [5b]$$

To properly record a signal from a physical source, the sampling rate ( $f_s$ ) has to be properly defined. The sampling rate is determined by the Nyquist frequency

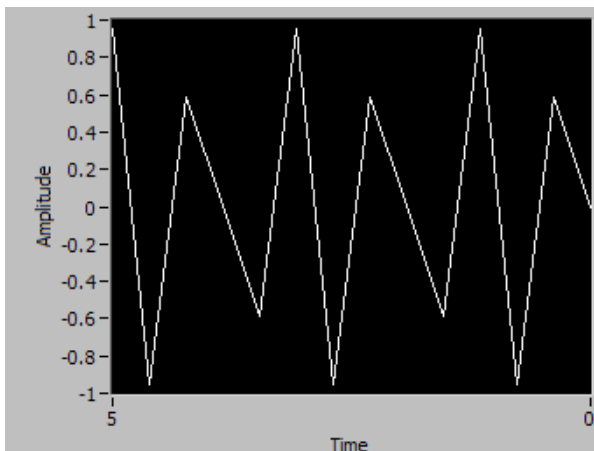
$$f_{Nyq} = \frac{f_s}{2} = \frac{1}{2\Delta t} \quad [6]$$

A spectrum is accurately sampled with frequencies lower than  $f_{Nyq} = f_s / 2$  and is not accurately sampled with frequencies greater than or equal to  $f_{Nyq} = f_s / 2$ . As an example, a simple sine waveform wit frequency  $f=1\text{Hz}$  and amplitude  $a=1$  may be recorded as follows (Figure 7) when different sampling rates are used. It is observed that what we “see” on computer screens (Figures 7a and b) is not what the signals are supposed to be due to the use of incorrect sampling rates.

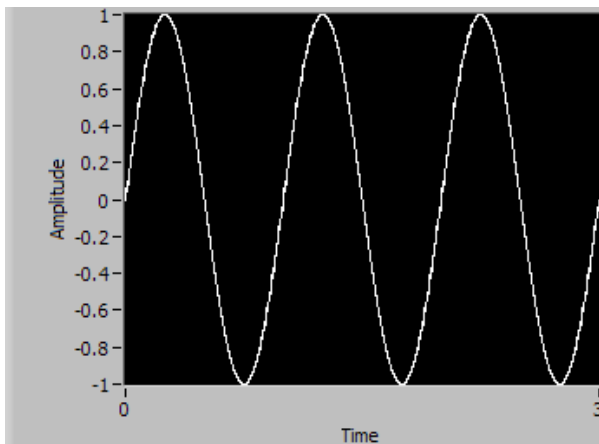




$$f_{Nyq} = 1 \text{ Hz}$$



$$f_{Nyq} = 1.25 \text{ Hz}$$



$$f_{Nyq} = 10 \text{ Hz}$$

**Figure 8 – Labview Front Panels showing the effect of sampling rate.**

### CONCLUSIONS

The interactive virtual instruments (Labview) have been brought to the classroom for teaching spectral analysis. The program is simple to use and allows students to “build and play” various

signals and instantaneously “see” the outcomes. The students learn the concepts through both rigorous mathematical developments and virtual simulations at the same time. In comparing with the traditional pure lecture-based method, this integrated approach involves both instructors and students in the teaching process.

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