

The Use of Symbolic Solvers in Engineering Education

R. Luck¹ and B. K. Hodge²

Abstract – There are many books and pedagogical papers on how to use mathematical CAD programs to perform numerical calculations for engineering problems. In contrast to numerical calculations, this paper explores the use of symbolic solvers for mechanical engineering problems and investigates the pedagogical inferences of using these solvers in mechanical engineering education. Classroom explanations and homework for engineering courses sometimes require tedious symbolic manipulations of equations or systems of equations, differentiation of complex functions, and evaluation of integral expressions. The use of computational software systems, such as Mathcad, Mathematica, Maple, and Macsyma, capable of symbolic manipulations, allows the student to focus more on the engineering aspects of a problem than on performing the algebra. With less time spent on evaluating integrals, performing complex differentiations, and solving systems of equations, more time is available for students to engage in higher-level synthesis and understanding. Several examples are presented in this paper to demonstrate how symbolic manipulation software can be successfully employed in the classroom and in homework. The examples are taken from both undergraduate and graduate courses. Although the examples in this paper are appropriate for mechanical engineering, the paradigm is transferable to any engineering discipline in which problem formulations result in systems of complex equations whose solutions require tedious (and error prone) manipulations.

Keywords: Symbolic solvers, CAD, Mathcad, engineering education

INTRODUCTION

Advanced mathematical CAD software offers a great opportunity to enhance the educational experience in engineering courses. The authors' have been actively involved in using mathematical computer solvers in engineering education in order to inculcate a systematic approach to problem solving, e.g., see Hodge and Taylor [1], Hodge [2], and Hodge and Luck [3]. In this paper the authors describe a different aspect of advanced mathematical CAD software: the ability to perform symbolic manipulations. This latter feature of some mathematical CAD software can be exploited by engineering instructors to help emphasize conceptual thinking while reducing unnecessary frustration on the students due to long algebraic steps and other repetitive analytical calculations required in differentiation, integration, and minimization of mathematical formulas. With less time spent on evaluating integrals, performing complex differentiations, and solving systems of equations, more time is available for students to engage in higher-level synthesis and understanding. This article describes how, at the Mechanical Engineering (ME) Department at Mississippi State University (MSU), symbolic solvers are used several courses including the undergraduate level Engineering Analysis (EA), System Dynamics (SD), and Introduction to Vibrations and Controls (IVC) courses and the graduate level Convective Heat Transfer (CHT) course. The CAD software used in the Mechanical Engineering Department at Mississippi State University is Mathcad. The web site for the Mathcad vender is www.ptc.com. A general reference, in addition to Mathcad web-based information, is that of Brent [4].

One of the pitfalls of using symbolic solvers is that, without careful guidance, computer generated symbolic calculations can rapidly degenerate into large cumbersome expressions that are meaningless to the students. For example, the following long expression for the vibrations of an underdamped mass-spring-damper system

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$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{1}{s} \text{ invlaplace, } s \rightarrow \frac{\cosh\left(t\sqrt{\zeta^2\omega_n^2 - \omega_n^2}\right) \cdot e^{-\zeta t\omega_n} \sqrt{\zeta^2\omega_n^2 - \omega_n^2} - \sqrt{\zeta^2\omega_n^2 - \omega_n^2} + \zeta\omega_n \cdot \sinh\left(t\sqrt{\zeta^2\omega_n^2 - \omega_n^2}\right) \cdot e^{-\zeta t\omega_n}}{\sqrt{\omega_n^2(\zeta^2 - 1)}}$$

can be greatly simplified by clearly stating the assumption on the variables, substituting for well known groups of variables, and factorizing terms of interest as follows:

$$\begin{array}{l} \text{assume, } \omega_n > 0 \\ \text{assume, } 0 < \zeta < 1 \\ \text{invlaplace, } s \\ \text{substitute, } \omega_n \sqrt{1 - \zeta^2} = \omega_d \\ \text{expand} \\ \text{collect, } e^{-\zeta t\omega_n} \end{array} \rightarrow \left(-\cos(t\omega_d) - \frac{\zeta \sin(t\omega_d)}{\sqrt{1 - \zeta^2}} \right) \cdot e^{-\zeta t\omega_n} + 1$$

Thus, the students need to be carefully guided on how to use symbolic manipulations in a productive manner. Without such guidance, students often “win” the solution battle, but lose the “understanding” war.

The following sections describe the philosophy behind the courses and present details and examples of typical homework.

PHILOSOPHY BEHIND THE ENGINEERING ANALYSIS (EA) COURSE

A main objective behind the EA course is to consolidate the mathematical skills acquired during the freshman and sophomore years while emphasizing their use in posing and solving appropriate engineering problems. This is partly accomplished through the process of learning how to use Mathcad. Students are introduced to Mathcad functionality by demonstrating how to define functions, symbolically perform differentiation and integration, simplify systems of algebraic and trigonometric expressions, perform series analysis, find optimal solutions, fit equations to data, and solve differential equations. In the homework problem illustrated in Figure 1, the objective is to guide the students to enhance their experience on the how to use Taylor series approximations. Figure 1 displays the solution posted by the instructor. Solutions provided by the students follow the same format and uses similar symbolic commands. As demonstrated in the example, a rather cumbersome expression is defined and graphed, then a third-order Taylor series approximation about the point $x = 1$ is obtained symbolically and compared in the same plot. Since the objective in this assignment is not to practice differentiation, but to clearly understand how well the Taylor series approximation works and the meaning of truncation error, the symbolic approach is well suited for this problem. Students are encouraged to change the order of the approximation by simply modifying one number in the symbolic Taylor series expansion command and observe how the graphs are automatically redrawn showing the possible improvements in the approximation; this is a great advantage of using symbolic solutions to interactively observe changes due to different parameter values or different functions. Students in the EA course responded well to this assignment and exhibited a good level of understanding on the value of Taylor series approximation and truncation error on a subsequent quiz.

PHILOSOPHY BEHIND THE SYSTEM DYNAMICS (SD) COURSE

The SD course is an introduction to modeling and understanding the dynamic response of systems typically encountered in mechanical engineering through the use of lumped parameters. These systems considered consist of combinations of mechanical, electrical, hydraulic, and pneumatic systems. Students are taught to draw a diagram of the system, e.g., a “free-body diagram” for the mechanical part of the system or an electric diagram with well defined current and voltage variables for the electrical part of the system, and to use the diagrams to obtain a set of equations modeling the transient or dynamic response of the system. More importantly, students are taught to interpret the differential equations modeling the transient response of the system from the perspective of time constants, damping ratios, and natural frequencies. The homework problem, presented in Figure 2, is designed to help the student understand the convenience of using the method of partial fraction expansions and how the time constants are readily available from each partial fraction. Figure 2 is representative of a homework turned in by a student and then used by the instructor as a solution posted in the course website. Symbolic calculations are used to simplify the algebra and the numerical substitutions allowing the students to concentrate on the form of the partial

fraction expansion, i.e., on the sum of the partial fractions. The verification process used a simple symbolic calculation to provide another solution that can be used to compare the results from that of the partial fraction solution.

PHILOSOPHY BEHIND THE INTRODUCTION TO VIBRATIONS AND CONTROLS (IVC) COURSE

The IVC course concentrates on vibrations and controls application using the mathematical modeling techniques learned in the SD course. One of the main objectives of the IVC course is to obtain a basic, engineering-level understanding of feedback control. A second objective is to become familiar with techniques used in vibrations of practical mechanical systems, for instance, vibration isolation and transmissibility of vibrations. A typical problem in introductory controls courses is to simplify a block diagram. The example in Figure 3, demonstrates how this tedious algebraic process can be easily tackled with a symbolic command. Figure 3 is also representative of a homework turned in by a student and then used by the instructor as a solution posted in the course website. The limited pedagogical information provided in the homework solution described in Figure 3 is representative of the typical homework turned in by students. The advantage of the symbolic commands is that they permit students to concentrate their efforts on posing the correct equations for subsequent algebraic reduction. This is one of the main advantages of using symbolic solvers in engineering education: allowing the students to concentrate their efforts in posing the problems correctly instead of spending most of their efforts in algebraic exercises. Instructors are then able to ask the students to tackle much more interesting problems that would otherwise require too much work by the students.

PHILOSOPHY BEHIND THE CONVECTION (GRADUATE) COURSE

The CHT course is a conventional graduate-level heat transfer course that emphasizes differential equation formulations and solutions. Much of the first part of the course is devoted to the classical laminar solutions of convective heat transfer. The textbook used is Bejan's *Convection Heat Transfer* (third edition) [5]. Mathcad is adopted to obtain the numerical solutions of virtually all the classical two-dimensional, constant-property, laminar convective heat transfer boundary-value problems, including free convection formulations. Experience in teaching the CHT course has demonstrated the pedagogical usefulness of providing solution procedures rather than just referring to tabular and/or graphical solution presentations. Class time is used to develop in detail (from the outline provided in the textbook) the general formulation of integral solutions to both the momentum and energy equations. Symbolic manipulation via Mathcad is utilized to demonstrate the simplest of the integral profile approximations—linear. A homework assignment is made requiring the students to do the complete integral solutions for a more complex profile assumption—quadratic, cubic, quartic, or trigonometric. Figure 4 is a copy of the Mathcad worksheet used to discuss the integral procedure with the linear profile used. Figure 5, the integral solution for a cubic profile, illustrates what a student is expected to accomplish as a homework assignment. The development of integral solutions for Prandtl numbers greater and less than unity forces to student to understand Bejan's scale analysis and how that translates into an understanding of the effect of Prandtl number on the resulting solutions. Experience indicates this to be an effective teaching/learning tool at the graduate level.

PEDAGOGICAL INFERENCES AND CONCLUSIONS

This paper presents examples of using a symbolic manipulation software element (from Mathcad in this paper) for aiding in the symbolic solutions of meaningful mechanical engineering problems. In all the examples explored in this paper, the same process is used. The treatments of all the example problems are identical and emphasize the three steps: (1) formulate a well-posed system of equations, (2) utilize a symbolic manipulator to do the "algebra," and (3) verify/explain the results. In this paper, the symbolic manipulation has been accomplished using Mathcad. Other computational software systems (Mathematics, Matlab,...) offer the same capability, albeit in different formats, but with the same results.

Anecdotally, students appreciate the reduction of tedious algebra using symbolic manipulation software. With less time spent on algebra, more time is available for students to engage in higher-level synthesis and understanding. Examples illustrating this approach to solutions of engineering problems in four courses (three undergraduate and one graduate) in ME at MSU have been presented and discussed. The approach offers advantages in providing

students with capability to solve more “real world” problems while concentrating on the engineering aspects of the problems.

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Given : $f(x) := x^2 \cdot e^{-x} \cdot \sin(x)$

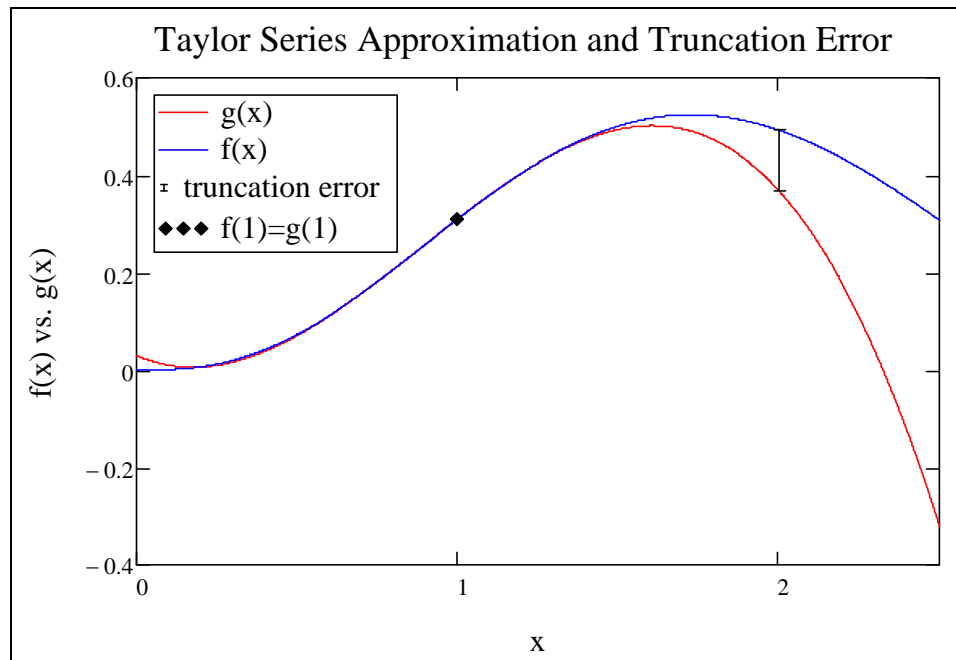
Find : The Taylor series expansion to the third derivative term to approximate the function about the point $x_0 = 1$.

Plot the function and the approximate function in a single graph and show the truncation when $x=2$.

Solution :

Three decimal points will be used in the results (float 3) to make the answers shorter.

<code>g(x) := f(x)</code>	<code>series, x = 1, 4</code>
	<code>float, 3</code>
	$\rightarrow 0.31 + (0.508 \cdot x - 0.508) - 0.111 \cdot (x - 1.0)^2 - 0.339 \cdot (x - 1.0)^3$



The truncation error at $x=2$ is the difference between both plots when $x=2$.

Figure 1. Worksheet for Example 1

Given:

The following transfer function: $TF(s) = \frac{y(s)}{u(s)} = \frac{(s + 50)}{s^2 + 12 \cdot s + 20}$

Find:

- (A) The response $y(t)$ to a unit step input using the method of partial fraction expansion.
 (B) The time $y(t)$ will take to reach within 1% of the steady-state value.

Solution:

Simplifying for the output of the system:

$$TF(s) := \frac{(s + 50)}{s^2 + 12 \cdot s + 20} \quad u(s) := \frac{1}{s} \quad Y(s) := TF(s) \cdot u(s) \text{ simplify } \rightarrow \frac{s + 50}{s \cdot (s + 2) \cdot (s + 10)}$$

it follows by partial fraction expansion that

$$Y(s) = \frac{A}{s + 10} + \frac{B}{s + 2} + \frac{C}{s} \quad \text{and the time solution is of the form} \quad y(t) = A \cdot e^{-10t} + B \cdot e^{-2t} + C$$

Solving for A, B, and C:

$$A := (s + 10) \cdot Y(s) \text{ substitute, } s = -10 \rightarrow \frac{1}{2}$$

$$B := [(s + 2) \cdot Y(s)] \text{ substitute, } s = -2 \rightarrow -3$$

$$C := s \cdot Y(s) \text{ substitute, } s = 0 \rightarrow \frac{5}{2}$$

Solution A

$$y(t) := \frac{1}{2} \cdot e^{-10 \cdot t} - 3e^{-2 \cdot t} + \frac{5}{2}$$

Part (B)

For the exponential component, e^{-10t} : $\tau_1 := \frac{1}{10}$ seconds

For the exponential component, $e^{-2 \cdot t}$: $\tau_2 := \frac{1}{2}$ seconds

Since, $\tau_2 > \tau_1$ then, τ_2 is the dominant root.

The time it will take for $y(t)$ to reach within 1% of the steady-state value:

$$t_{\text{steadystate}} := 4.6 \cdot \tau_2 \cdot s$$

Solution B

$$t_{\text{steadystate}} = 2.3 \text{ s}$$

Verification:

For part (A), the results are verified by using the basic inverse Laplace symbolic function to obtain the time response of the transfer function give in the problem statement. The compared time responses are identical, therefore the results are verified.

$$y(t) := Y(s) \text{ invlaplace, } s \rightarrow \frac{e^{-10 \cdot t}}{2} - 3 \cdot e^{-2 \cdot t} + \frac{5}{2}$$

For part (B), the time response $y(t)$ of the system is plotted versus time (t). Upon visual inspection, it can be seen that at approximately 4.6 x time constant (2.3 seconds) the system has reached within 1% to the steady-state value.

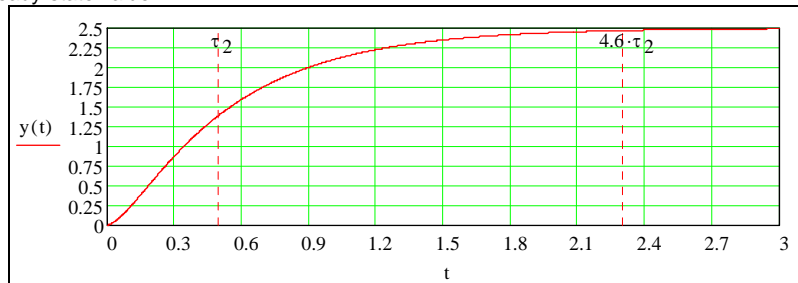
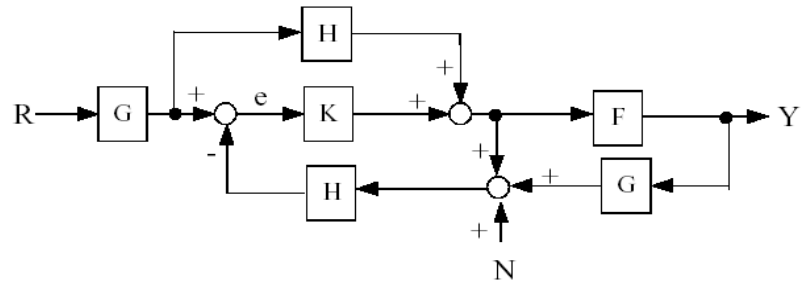


Figure 2. Worksheet for Example 2

Given: The block diagram shown



Find: The transfer function relating the output e to the input R .

Solution:

All inputs other than R must be set to zero: $N = 0$

Given

$$Y = F \cdot (e \cdot K + R \cdot G_1 \cdot H_1)$$

$$e = R \cdot G_1 - H_2 \cdot (e \cdot K + R \cdot G_1 \cdot H_1 + Y \cdot G_2)$$

$$\text{Find}(Y, e) \rightarrow \left(\begin{array}{l} \frac{F \cdot G_1 \cdot H_1 \cdot R + F \cdot G_1 \cdot K \cdot R}{H_2 \cdot K + F \cdot G_2 \cdot H_2 \cdot K + 1} \\ -\frac{G_1 \cdot H_1 \cdot H_2 \cdot R - G_1 \cdot R + F \cdot G_1 \cdot G_2 \cdot H_1 \cdot H_2 \cdot R}{H_2 \cdot K + F \cdot G_2 \cdot H_2 \cdot K + 1} \end{array} \right)$$

$$\frac{e}{R} = -G_1 \cdot \frac{(-1 + H_2 \cdot G_2 \cdot F \cdot H_1 + H_2 \cdot H_1)}{(1 + H_2 \cdot G_2 \cdot F \cdot K + H_2 \cdot K)}$$

Figure 3. Worksheet for Example 3

Plot of velocity profile shape for various functions:

$m_{lin}(n) := n$ linear profile

$m_{cub}(n) := \frac{n}{2} \cdot (3 - n^2)$ cubic profile

$m_{sin}(n) := \sin\left(\pi \cdot \frac{n}{2}\right)$ sine profile

$m_{quar}(n) := 2 \cdot n - 2 \cdot n^3 + n^4$ quartic profile

Taking the Blasius solution as the exact solution.

$i := 0..100$

$n_i := \frac{i}{100}$

IC := $\begin{pmatrix} 0 \\ 0 \\ 0.4696 \end{pmatrix}$

$d(x,y) := \begin{pmatrix} y_1 \\ y_2 \\ -y_0 \cdot y_2 \end{pmatrix}$

Vector of initial conditions.

Definition of Blasius equation as three first-order DEs.

Invoke **rkfixed** to solve the Blasius equation as well as δ^* and θ .

$z := rkfixed(IC, 0, 8, 80, d)$ Fixed-step Runge-Kutta solution.

The array, **z**, contains the results with the independent variable as the first column.

$ii := 0, 1..80$ range variable

$\eta_{ii} := (z^{(0)})_{ii}$ η , the independent variable

$f_{ii} := (z^{(1)})_{ii}$ $f(\eta)$ $fp_{ii} := (z^{(2)})_{ii}$ $f'(\eta)$ $fp_{35} = 0.991$ $\eta_{35} = 3.5$

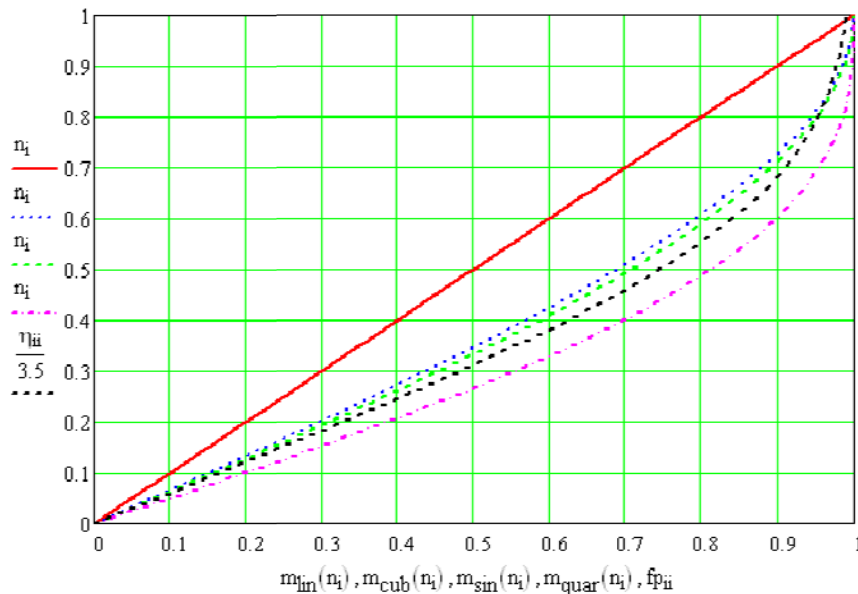


Figure 4. Worksheet for Example 4

Integral Method for Laminar Boundary-Layer Flow.

Linear velocity profile:

$$m(n) := n$$

$$dm/dn(n) := \frac{d}{dn} m(n) \quad dm/dn(0) = 1$$

$$\delta_{sint} := \int_0^1 (1 - m(n)) \, dn \quad \delta_{sint} = 0.5$$

$$\theta_{int} := \int_0^1 m(n) \cdot (1 - m(n)) \, dn \quad \theta_{int} = 0.167$$

$$\frac{H}{\delta_{sint}} := \frac{\delta_{sint}}{\theta_{int}} \quad H = 3$$

$$a_1 := \left[\frac{2 \cdot dm/dn(0)}{\int_0^1 m(n) \cdot (1 - m(n)) \, dn} \right]^{-0.5} \quad a_1 = 3.464$$

$$a_2 := \left[(2 \cdot dm/dn(0)) \cdot \int_0^1 m(n) \cdot (1 - m(n)) \, dn \right]^{-0.5} \quad a_2 = 0.577$$

$$\int_0^1 m(p \cdot \text{del}) \cdot (1 - m(p)) \, dp \rightarrow \frac{\text{del}}{6}$$

Solve for "del."

$$\text{Pr} = 2 \cdot \frac{dm/dn(0)}{(\text{del} \cdot a_1)^2 \cdot \left(\frac{1}{6} \cdot \text{del}\right)} \left[\begin{array}{c} \frac{1}{\text{Pr} \cdot a_1} \cdot 12^{\frac{1}{3}} \cdot \left(dm/dn(0) \cdot \text{Pr}^2 \cdot a_1 \right)^{\frac{1}{3}} \\ \frac{-1}{2 \cdot \text{Pr} \cdot a_1} \cdot 12^{\frac{1}{3}} \cdot \left(dm/dn(0) \cdot \text{Pr}^2 \cdot a_1 \right)^{\frac{1}{3}} + \frac{1}{2} \cdot \frac{3^{\frac{1}{2}}}{\text{Pr} \cdot a_1} \cdot 12^{\frac{1}{3}} \cdot \left(dm/dn(0) \cdot \text{Pr}^2 \cdot a_1 \right)^{\frac{1}{3}} \\ \frac{-1}{2 \cdot \text{Pr} \cdot a_1} \cdot 12^{\frac{1}{3}} \cdot \left(dm/dn(0) \cdot \text{Pr}^2 \cdot a_1 \right)^{\frac{1}{3}} - \frac{1}{2} \cdot \frac{3^{\frac{1}{2}}}{\text{Pr} \cdot a_1} \cdot 12^{\frac{1}{3}} \cdot \left(dm/dn(0) \cdot \text{Pr}^2 \cdot a_1 \right)^{\frac{1}{3}} \end{array} \right]$$

$$\text{value} := \frac{1}{a_1} \cdot \sqrt[3]{12} \cdot \left(dm/dn(0) \cdot a_1 \right)^{\frac{1}{3}} \quad \text{value} = 1 \quad \text{del} = \frac{1}{\text{Pr}^{0.333}}$$

$$N_{\text{value}} := \frac{dm/dn(0)}{\text{value} \cdot a_1} \quad N_{\text{value}} = 0.289 \quad \text{So that} \quad Nu_x = 0.289 \cdot Re_x^{0.5} \cdot \text{Pr}^{0.333}$$

Figure 4. Continued

Cubic velocity profile:

$$m(n) := \frac{n}{2} (3 - n^2)$$

$$\frac{dm}{dn}(n) := \frac{d}{dn} m(n) \quad \frac{dm}{dn}(0) = 1.5$$

$$\delta_{\text{int}} := \int_0^1 (1 - m(n)) \, dn \quad \delta_{\text{int}} = 0.375$$

$$\theta_{\text{int}} := \int_0^1 m(n) \cdot (1 - m(n)) \, dn \quad \theta_{\text{int}} = 0.139$$

$$H := \frac{\delta_{\text{int}}}{\theta_{\text{int}}} \quad H = 2.692$$

$$a_1 := \left[\frac{2 \cdot \frac{dm}{dn}(0)}{\int_0^1 m(n) \cdot (1 - m(n)) \, dn} \right]^{0.5} \quad a_1 = 4.641$$

$$a_2 := \left[(2 \cdot \frac{dm}{dn}(0)) \cdot \int_0^1 m(n) \cdot (1 - m(n)) \, dn \right]^{-0.5} \quad a_2 = 0.646$$

$$\int_0^1 m(p \cdot \text{del}) \cdot (1 - m(p)) \, dp \rightarrow -\frac{3 \cdot \text{del} \cdot (\text{del}^2 - 14)}{280}$$

$$\text{Pr} = 2 \cdot \frac{\frac{dm}{dn}(0)}{(\text{del} \cdot a_1)^2 \cdot \left(\frac{3}{20} \cdot \text{del}\right)} \left[\frac{2}{3 \cdot \text{Pr} \cdot a_1} \cdot \sqrt[3]{45} \cdot \left(\frac{dm}{dn}(0) \cdot \text{Pr}^2 \cdot a_1\right)^{\frac{1}{3}} \right]$$

$$\text{value} := \frac{2}{3 \cdot a_1} \cdot \sqrt[3]{45} \cdot \left(\frac{dm}{dn}(0) \cdot a_1\right)^{\frac{1}{3}} \quad \text{value} = 0.976$$

$$\text{del} = \frac{0.976}{\text{Pr}} \quad \text{Nvalue} := \frac{\frac{dm}{dn}(0)}{\text{value} \cdot a_1} \quad \text{Nvalue} = 0.331$$

So that $\text{Nu}_x = 0.331 \cdot \text{Re}_x^{0.5} \cdot \text{Pr}^{0.333}$

Figure 5. Cubic profile solution worksheet

Solution for low Pr fluids:

$$\int_0^{\frac{1}{\text{del}}} m(p \cdot \text{del}) \cdot (1 - m(p)) dp \rightarrow \frac{175 \cdot \text{del}^3 - 168 \cdot \text{del}^2 + 32}{280 \cdot \text{del}^4}$$

$$\int_{\frac{1}{\text{del}}}^1 (1 - m(p)) dp \rightarrow \frac{(\text{del} - 1)^3 \cdot (3 \cdot \text{del} + 1)}{8 \cdot \text{del}^4}$$

$$\int_0^{\frac{1}{\text{del}}} m(p \cdot \text{del}) \cdot (1 - m(p)) dp + \int_{\frac{1}{\text{del}}}^1 (1 - m(p)) dp \rightarrow \frac{175 \cdot \text{del}^3 - 168 \cdot \text{del}^2 + 32}{280 \cdot \text{del}^4} + \frac{(\text{del} - 1)^3 \cdot (3 \cdot \text{del} + 1)}{8 \cdot \text{del}^4}$$

$$\text{Pr} = 2 \cdot \frac{\text{dmdn}(0)}{(\text{del} \cdot a_1)^2 \cdot \left(\frac{1}{280} \cdot \frac{32 - 168 \cdot \text{del}^2 + 175 \cdot \text{del}^3}{\text{del}^4} + \frac{3}{8} - \frac{1}{8} \cdot \frac{8 \cdot \text{del}^3 + 1 - 6 \cdot \text{del}^2}{\text{del}^4} \right)}$$

$\text{Pr} := 0.00001$ $\Delta := 1$ Pr specification and initial guess for del.

Given

$$\text{Pr} = 2 \cdot \frac{\text{dmdn}(0)}{(\Delta \cdot a_1)^2 \cdot \left(\frac{1}{280} \cdot \frac{32 - 168 \cdot \Delta^2 + 175 \cdot \Delta^3}{\Delta^4} + \frac{3}{8} - \frac{1}{8} \cdot \frac{8 \cdot \Delta^3 + 1 - 6 \cdot \Delta^2}{\Delta^4} \right)}$$

$\Delta := \text{Find}(\Delta)$ $\Delta = 193.224$ $\text{number} := \Delta \cdot \text{Pr}^{0.5}$ $\text{number} = 0.611$

$\text{Nvalue} := \frac{\text{dmdn}(0)}{\text{number} \cdot a_1}$ $\text{Nvalue} = 0.529$

Summary of Pr versus Δ results:

Pr	Δ	number	Nvalue
0.1	2.389	0.756	0.428
0.01	6.582	0.658	0.491
0.001	19.769	0.625	0.517
0.0001	61.444	0.614	0.526
0.00001	193.233	0.611	0.529

Figure 5. Concluded

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He received the B.S. degree from Texas Tech University in 1984, and the M.S. and Ph.D. degrees from Penn State Univ., University Park, in 1987 and 1989, respectively, all in Mechanical Engineering. In 1989 he joined the faculty at the Mechanical Engineering Dept. at Mississippi State University. His current research interest is in the area of simulation, optimization and control of building cooling heating and power. He has published in the areas of building cooling heating and power, uncertainty analysis, inverse heat transfer, radiation heat transfer, applied math, theoretical and applied control systems, piezoelectric sensors, electrical power system generation and distribution, and redundant measurement systems.

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