

## **Student Projects to Improve Student Proficiency with Three-Dimensional Vectors**

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### **Abstract**

The Entrepreneurial Mindset (EM) asks students to use their curiosity to make connections and create value for stakeholders and ultimately the world. We introduced these tenets into Engineering Statics during Fall 2021 using a card published on Engineering Unleashed by Glenn Gaudette and Sarah Wodin-Schwartz and adapted to large classes following the methods of McCandless and Howard.<sup>1-2</sup> This paper investigates the effect of this team project on student performance on three-dimensional particle equilibrium. We compared midterm exam 1 in Fall 2020 to those from Fall 2019 and Fall 2020 to assess student ability to convert from direction cosines and projection in a plane to Cartesian form. We also tested student understanding of reaction forces for free-body diagrams in 3D. We found small difference that were not statistically significant, but anecdotally we believe that the inclusion of the team project increased student understanding.

### **Keywords**

Statics, three dimensional vectors, free body diagrams, student projects, entrepreneurial mindset

### **Introduction**

The Entrepreneurial Mindset (EM) promoted by the Keen Foundation and the Engineering Unleashed community encourages students to exercise their curiosity, to make connections between their courses and the real world, and to learn to create value for their companies by thinking outside the box.<sup>3</sup> This framework is often included into the curriculum by using ill-defined projects and other active learning techniques.<sup>4</sup> Integration of (EM) in university curriculum can benefit not only students on individual (metacognitive) level but also on a social (encouragement in local/regional entrepreneurship) level.<sup>5</sup> At NC State we are introducing these ideas into multiple courses across the Mechanical Engineering curriculum.

Engineering Statics at NC State has been taught as a flipped class since 2011. Two team projects were introduced in the beginning and later-half of the Statics coursework. Ill-defined problems do not have a single solution. These projects include design choices and socioeconomic impacts that are less-commonly addressed in a classroom. This paper analyzes the student results from the first of these projects.

Statics at NC State begins with particle equilibrium and includes three-dimensional problems. Students struggle to create any connection between the math and the balancing act that is equilibrium once three dimensions come into play. Even when they can do all the math to solve three equations in three unknowns, student skills for visualization in 3D limit their conceptual learning. Additionally, the increase in algebra involved becomes an impediment to connecting

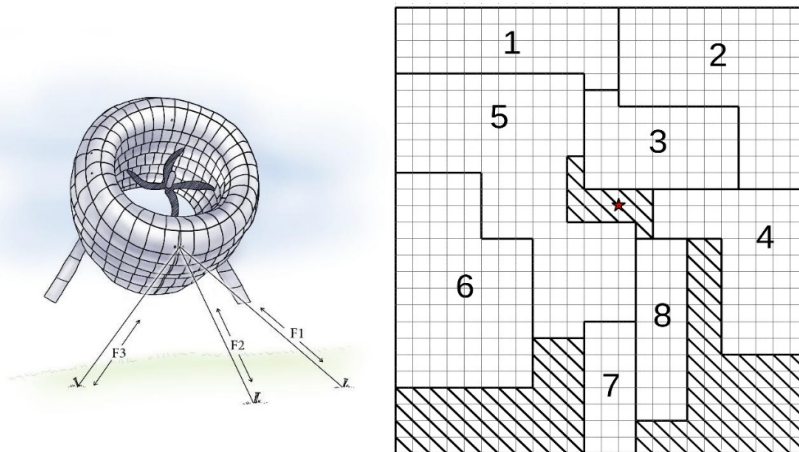
what’s happening in the world with what’s happening on the paper. The Flying Forces card was adapted for use in this class.<sup>1</sup> Our goal for this project was to help students connect 3D forces and particle equilibrium to a real-world scenario.

We assessed the students’ capability to solve 3D particle equilibrium and understanding of 3D force vectors before and after introducing the EM framework assignment into the coursework using the first midterm exam which is given at the end of the third week of class. Calculus-based Physics (Physics I) and Calculus II are prerequisites to this course, so students should have some conceptual idea about 3D vectors. However, students tend to struggle with these concepts on exams. The most common errors regarding these questions are: (1) Absence of a force on the body when it should be there, (2) Putting a force on the body even though it is acting on the body through another part that is also present in the diagram (double marking) and (3) Putting a force on the body although it doesn’t act on the body. The answer choices for the questions asked in the exam entertain some choices that include the errors mentioned above.

### The Balloon Project

The students were asked to design a cable tethering system for a balloon turbine to be suspended mid-air. (See Figure 1.) The external force acting on the turbine was given as a function of height. Student teams were asked to choose three anchor locations in the given zones shown in Figure 1. Each zone came with a description of the social impact and construction cost if that zone was chosen to hoist the turbine. A single zone could not anchor all three tethers. Three cable choices were provided with ultimate tension values: student teams chose to optimize cost, social impact, or power production. Teams drew free-body diagrams based on their designs and worked out the tensions in the cables reinforcing their 3D particle equilibrium learning.

Figure 1: Figure 1. Flying Balloon<sup>1</sup> and Field Map<sup>6</sup>



Students submitted their design in six separate parts. After the first day of class, students individually reflected on why anyone would want such a turbine including the engineering design choices and their overall social and economic impact. On Day 5 they are asked to form a team and reflect upon how they are going to approach the problem *as a team* using the assigned team contract.<sup>6</sup> On Day 6 student teams submitted their initial designs: the students were asked to evaluate the forces in the tethers and verify whether the tethers would break based on their

choice of material. Students were also asked to calculate payback time based on the material costs, power generation rate and the revenue from power sold to neighboring communities. On Day 7 students submitted a design report and an optimized solution. In both the Day 6 and Day 7 submissions, students were asked to give their cable locations in three different ways, one for each cable: as a magnitude along a line, using direction cosines, or using a projection into a plane. More details about this project and how it was incorporated in the coursework are given in McCandless and Howard.<sup>2</sup>

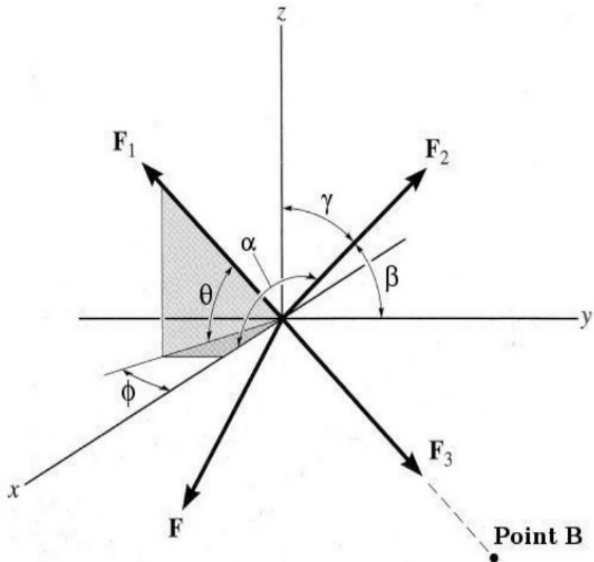
The first exam is conducted right after finishing up particle equilibrium. The student learning objectives for this exam include expressing forces into Cartesian form, identifying correct 3D FBDs, and solve equilibrium problems in three dimensions.

### Force Projection in a Plane

One common type of exam question is to ask students to express in Cartesian form a force presented using direction cosine angles, projection in a plane angles, or as a magnitude along a line. (See Figure 2) The students were asked to write the force vector or choose the correct answer in Cartesian form. Exam questions are shown in figures 2-4 for Fall 2019 – Fall 2021. Student performance for 3D vectors are compared using fall semesters only since prior research has shown that spring semesters are statistically different than fall.<sup>7</sup>

**Figure 2:** 3D Vectors Question from Exam 1, Fall 2019

Consider  $F_2$  with magnitude  $|F_2| = 315$  lb. The angle between  $F_2$  and the positive y axis is  $\beta = 40$  degrees. The angle between  $F_2$  and the positive z axis is  $\gamma = 60$  degrees. Express  $F_2$  in Cartesian form.



Select one:

- a.  $241 j + 157 k$  lb
- b.  $127 i + 241 j + 158 k$  lb
- c.  $51.4 i + 241 j + 158 k$  lb
- d.  $121 i + 209 j + 273 k$  lb
- e.  $209 j + 273 k$  lb
- f.  $209 i - 121 j + 273 k$  lb

Figure 3: 3D Vectors Question from Exam 1, Fall 2020

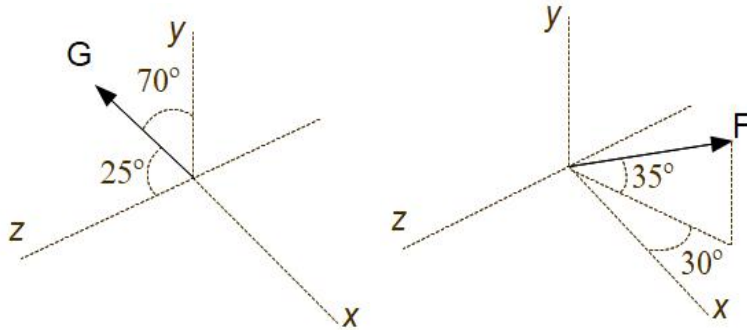


Figure 4: 3D Vectors Question from Exam 1, Fall 2021

Express each of these vectors in Cartesian form.

35 lb force =   $\hat{i}$  +   $\hat{j}$  +   $\hat{k}$  lb

36 lb force =   $\hat{i}$  +   $\hat{j}$  +   $\hat{k}$  lb

37 lb force =   $\hat{i}$  +   $\hat{j}$  +   $\hat{k}$  lb

### Free Body Diagrams

In other problems, students are asked to identify a correct 3D Particle equilibrium free-body diagram. Like virtually all 3D particle equilibrium problems, these questions were based on an object being suspended mid-air with cords. The principal learning objectives tested were whether students knew what reaction forces to add and whether they could identify which ropes had the same tensions and which did not. In Fall 2019 and 2020, the questions were multiple choice: choose the correct FBD from the five choices. In Fall 2021, students were given an FBD and asked to identify which loads should and should not be on the FBD. Questions were taken from textbooks and are available upon request.

### Results and Summary

Test 1 was taken by 355 students in Fall 2019, 372 students in Fall 2020, and 350 students in Fall 2021. Students were tested on whether they could change vectors from magnitude and direction to Cartesian form in 3D. In Fall 2019 this was a multiple choice question with only five choices. In Fall 2020 and Fall 2021 students had to fill in the blank or choose from many more options. For projection in a plane, results are separated into two questions: do the students identify the component correctly which does NOT use the projection, and do they correctly identify the components which do use the component. For example, in the second image in Figure 3, students

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can correctly calculate the y component as  $F \cdot \sin(35)$  whether the students have identified this force as defined with a projection in a plane or if they've misidentified it as using direction cosines. Finding this component is therefore easier than finding either the x or z components ( $F \cdot \cos(35) \cdot \cos(30)$  and  $F \cdot \cos(35) \cdot \sin(30)$  respectively). We therefore looked at whether the students got the more difficult components correct.

**Table 1:** Performance of Students (Percentage Correct) for Projection in a Plane

Projection in a Plane	Fall 2019	Fall 2020	Fall 2021
Both secondary components correct	81%	65%	47%
One secondary component correct but not both	14%	15%	25%
Neither secondary component correct	5%	20%	28%

Students did worse in Fall 2021 than in either of the prior semesters. However, it is impossible to separate whether that was because of the multiple-choice nature of the prior years vs the fill-in-the-blank question asked in Fall 2021. For example, in Fall 2019 identifying the primary component  $35 \cdot \sin(50)$  correct reduced the entire problem to a 50-50 chance of getting the problem correct. Getting the primary component correct did not aid getting the secondary components correct at all for a fill-in-the-blank problem. Also of course Fall 2020 was completely online during the COVID-19 pandemic.

For the direction cosines problem, the Fall 2020 cohort also had a fill-in-the-blank question. In all cases, the signs on the components have been neglected. For direction cosines there are two primary angles: referring again to Figure 3,  $\cos(25)$  in the z direction and  $\cos(70)$  in the y direction is considerably easier than using  $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$  to find the third angle. We neglected Fall 2019 as not comparable based on the fact that it was multiple choice. We compared the Fall 2020 and Fall 2021 data:

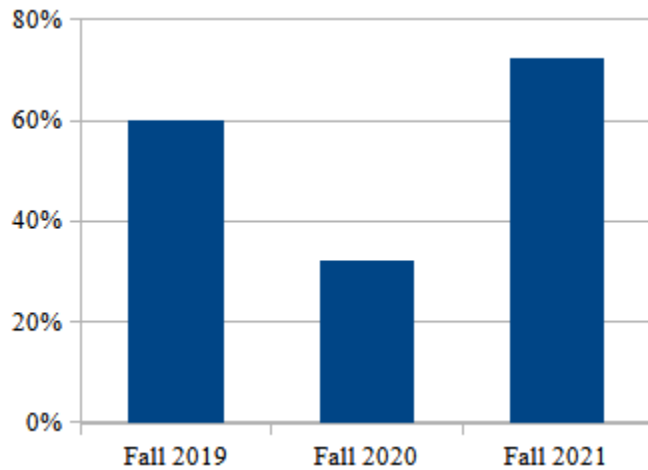
**Table 2:** Performance of Students (Percentage Correct) for Direction Cosines

Correctly Identified Direction Cosines	Fall 2020	Fall 2021
One primary but not both	12%	16%
Both primary	83%	70%
Secondary	63%	76%
All Correct	64%	41%

Students did worse during Fall 2021 than in Fall 2020. This finding was disappointing as students were back in the classroom in Fall 2021.

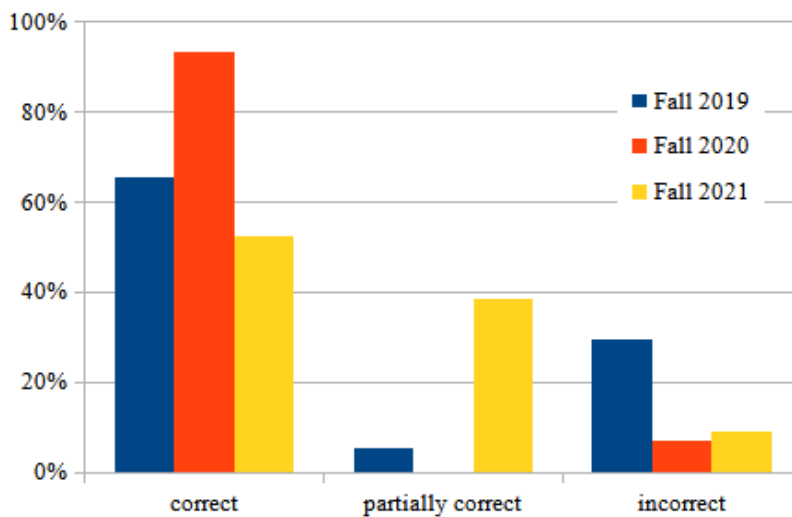
In the free-body diagram questions students were asked to identify what loads should go on the diagrams. We split the question into two possible errors: did students put unnecessary reaction forces on the free-body diagram and did they understand whether tensions in the cables should be the same or not. Figure 5 shows the percentage of students that chose the correct reaction forces in the FBD. In Fall 2019, 40% of students chose an option with  $R_x$ ,  $R_y$ , and  $R_z$  even though the point for the FBD was floating in space. In Fall 2021 students were 12% more likely than Fall 2020 to correctly leave off reaction forces and 40% better than the similar problem during the pandemic.

**Figure 5:** Percent of Student Correctly Identifying FBD Reactions



Students were also asked to identify the correct tensions on a 3D FBD. The exam question was easier in Fall 2020, but comparing Fall 2019 to 2021, the number of students getting the FBD incorrect dropped considerably.

**Figure 6:** Percent of Students Correctly Identifying Tensions on 3D FBD

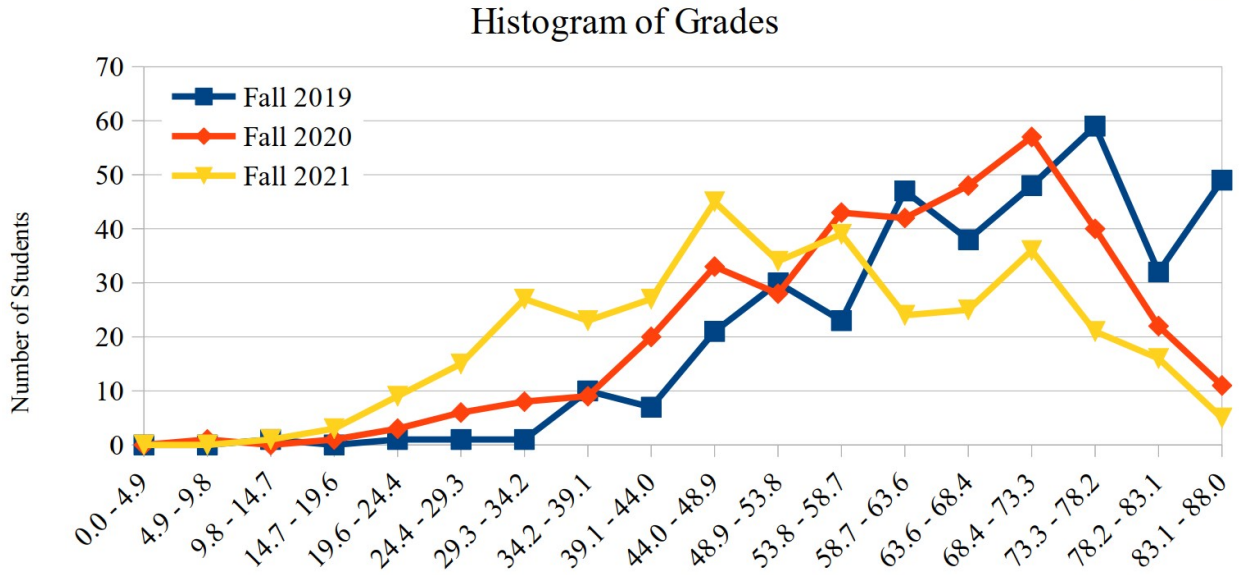


Identifying the fully correct tensions was significantly harder in fall 2021 because the exam allowed for considerably more variance in the answers.

**Discussion and Conclusions**

The results were disappointing and confounded by the pandemic. We hoped to show a clear improvement in student learning. Overall, the student performance in Fall 2020 and Fall 2021 have been disappointing. The Fall 2019 exam may have been easier with the multiple-choice questions, but the student performance was significantly better: Figure 6 shows the histogram of student scores for Fall 2019, 2020, and 2021. The students overall performed better before the pandemic. Student averages were 67.6, 61.3, and 53.4 out of 88 points over fall 2019, 2020, and 2021 respectively. It becomes difficult at that point to attribute any performance to anything other than the pandemic

**Figure 7:** Grade Histogram in Statics Before, During, and After the Pandemic



Future work will continue to explore how team projects such as this can help student learning once we have come out from the shadow of the global pandemic.

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