Using Mathcad to Enhance the Effectiveness of the Wind Energy Topic in an Alternate Energy Sources Course

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Abstract – The use of Mathcad to enhance the classroom presentation and extend the level of a meaningful quantitative experience for wind energy systems in an alternate energy systems survey course is explored. Because of time constraints, less than two weeks of class time is available for wind energy. One of the instructional goals for wind energy systems is to present a meaningful quantitative experience for the topic. By introducing Mathcad and developing a worksheet centered about the Weibull distribution as particularized to wind energy metrics, the quantitative experience has been extended to the evaluation of system performance—a significant and useful enhancement. The use of Mathcad to enhance the quantitative coverage of wind energy systems was a success.

Keywords: Wind turbine, Weibull distribution, power coefficient, Mathcad.

INTRODUCTION

The escalating cost of energy is a dominant feature of the 21st Century and has resulted in increased interest in alternate energy sources topics in engineering education. At Mississippi State University, ME 4353 Alternate Energy Sources [1, 2] is taught as a technical elective/beginning graduate course in the Mechanical Engineering program. The course is a survey course that examines a number of alternate energy sources and alternate applications of existing energy sources. One of the topics considered is wind energy. Because of time constraints, less than two weeks of class time is available for most topics, including wind energy. The instructional goals of each alternate energy topic are to present the relevant harvesting principle, to survey the availability of the source, and to present a meaningful quantitative experience for the topic. This paper explores the use of Mathcad to enhance the classroom presentation and extend the level of a meaningful quantitative experience for wind energy. The wind energy topic in earlier offerings of the course examined the taxonomy of wind energy systems [3], the availability of wind energy resources [4], and the role of the power coefficient in wind energy [3, 4] and presented some examples of available wind energy systems and some fundamental computations involving wind turbines [3, 4]. However, the computations, the "quantitative" experience, did not extend beyond routine calculations for specified conditions. By introducing Mathcad and developing a worksheet centered about the Weibull distribution as particularized to wind energy metrics, the quantitative experience has been extended to the evaluation of system performance—a significant and useful enhancement. Discussion and examples of the use of Mathcad in wind energy engineering topics in the course are presented in the following sections.

WIND SPEED DISTRIBUTION

Any assessment of wind energy topics requires a description of the wind speed distribution. The Weibull distribution is commonly used to assess the metrics of a wind turbine. The probability of occurrence of a given wind speed as expressed by the Weibull distribution [4] is

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$$h(v,k,c) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \tag{1}$$

where v is the wind speed, c is the scale parameter, and k is the shape parameter. The shape parameter controls the shape of the distribution; the larger the shape factor, the closer the distribution comes to Gaussian. The scale parameter controls the value of the mode (the most probable speed). The larger the scale parameter, the larger the mode and the smaller the probability of a given speed less than the mode. The shape parameter is dimensionless, and the scale parameter must have the same units as the speed. For typical wind distributions, the shape parameter value is usually near 2 [4].

Figure 1, generated using Mathcad, illustrates the probability distribution, expressed in hours per year per mph for scale parameters values of 10 mph, 15 mph, and 20 mph for a constant value of k = 2. The smaller the value of the scale parameter, the more hours at lower wind speeds. As the value of c increases, the mode wind speed values increase and the number of hours per year at wind speeds greater than the mode increases.
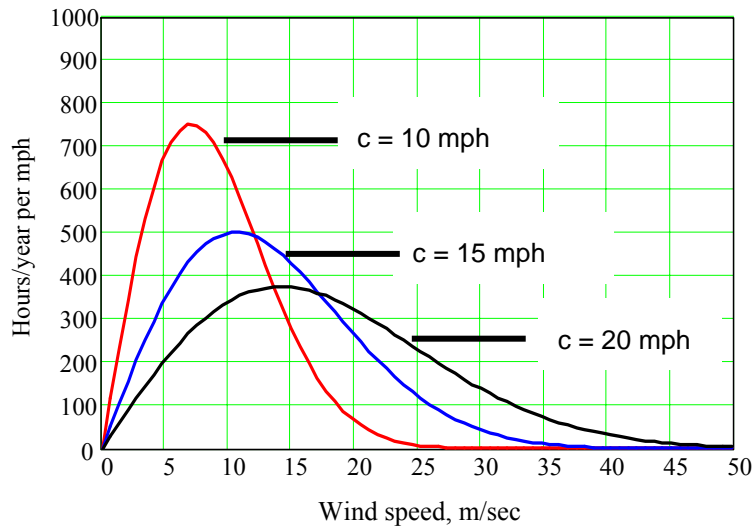

Figure 1. Weibull distribution for various c values and k = 2.

How does the Weibull distribution relate to assessing the metrics of wind energy? The mode speed is the most probable speed in a distribution. The mean speed is defined as

$$V_{mean} = \frac{1}{8760}\int_0^\infty h(v,k,c) \cdot v \cdot dv \tag{2}$$

Since the wind power (hp or kW) is proportional to the cube of the wind speed, the power density available for collection over a year per unit area of swept area is

$$Power_{avail} = \frac{1}{8760}\int_0^\infty \frac{1}{2} \cdot \rho \cdot h(v,k,c) \cdot v^3 \cdot dv \tag{3}$$

Thus, the speed of interest for wind energy is the root-mean-cube speed, V_{rmc}, defined as

$$V_{rmc} = \sqrt[3]{\frac{1}{8760}\int_0^\infty h(v,k,c) \cdot v^3 \cdot dv} \tag{4}$$

and the annual average power density available becomes

$$Power_{avail} = \frac{1}{2}\rho V_{rmc}^3 \tag{5}$$

WIND ENERGY FUNDAMENTALS

The power coefficient is defined as the power extracted divided by the available power of the wind stream or

$$Cp = \frac{Power_{ext}}{\frac{1}{2}\rho AV_{wind}^3} \tag{6}$$

The maximum value of the power coefficient, the Betz limit, is 0.5926 [3, 4]. The Betz limit represents the maximum value of the power coefficient and defines the maximum power that can be extracted from a given wind stream. The power coefficient is perhaps the most important single metric used in characterizing a wind turbine. A reasonable value of the power coefficient for a modern, well-designed wind turbine is 0.5. The annual average extraction power density can be cast as

$$Power_{ext} = \frac{1}{4}\rho V_{rmc}^3 \tag{7}$$

With Cp = 0.5, the total energy that can extracted per year for a given distribution is the integral of Eq. (6) for each velocity over all possible velocities or

$$Energy_{rmc} = 0.25 \cdot \rho \int_0^\infty h(v,k,c) \cdot 8760 \cdot v^3 \cdot dv \tag{8}$$

The best way to assimilate the aforementioned is to consider some example problems.

WIND ENERGY EXAMPLES

Example 1

Find V_{mode}, V_{mean}, V_{rmc}, the power density available distribution, and the power extracted per m^2 for a wind turbine at a site which possesses a Weibull wind distribution with c = 15 m/s and k = 1.5. The density is 1.225 kg/m^3.

Solution:
A graphical representation of the Weibull distribution for k = 1.5 and c = 15 m/sec is presented in Figure 2.
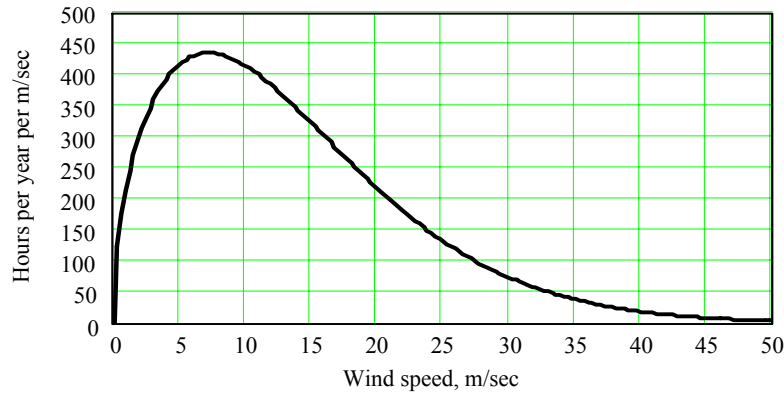

Figure 2. Weibull distribution for k = 1.5 and c = 15 m/sec.

The mode, the most probable wind speed, occurs at 7.21 m/sec. The mean wind speed and the root-mean-cube speed are defined in Eqs. (2) and (4), respectively. The arithmetic for this example is accomplished in Mathcad, and the Mathcad worksheet is reproduced in Figure 3.

$\rho := 1.225 \dfrac{kg}{m^3}$ density $mph := \dfrac{mi}{hr}$ define mph $kW := 1000 \, watt$ define kW

$c := 15 \cdot \dfrac{m}{sec}$ $k := 1.5$ specify values of the scale parameter and shape parameter

$h(v,k,c) := \dfrac{k}{c} \cdot \left(\dfrac{v}{c}\right)^{k-1} \cdot e^{-\left(\frac{v}{c}\right)^k}$ Weibull distribution function definition.

$PowerDen(V) := 0.5 \cdot \rho \cdot V^3$ Power density.

$V_{mode} := 7.21 \cdot \dfrac{m}{sec}$ $PowerDen(V_{mode}) = 229.563 \dfrac{watt}{m^2}$

$V_{mean} := \displaystyle\int_{0 \cdot \frac{m}{sec}}^{\infty \cdot \frac{m}{sec}} \dfrac{k}{c} \cdot \left(\dfrac{v}{c}\right)^{k-1} \cdot e^{-\left(\frac{v}{c}\right)^k} \cdot v \, dv$ $V_{mean} = 13.541 \dfrac{m}{sec}$ $PowerDen(V_{mean}) = 1.521 \times 10^3 \dfrac{watt}{m^2}$

$V_{rmc} := \sqrt[3]{\displaystyle\int_{0 \cdot \frac{m}{sec}}^{\infty \cdot \frac{m}{sec}} \dfrac{k}{c} \cdot \left(\dfrac{v}{c}\right)^{k-1} \cdot e^{-\left(\frac{v}{c}\right)^k} \cdot v^3 \, dv}$ $V_{rmc} = 18.899 \dfrac{m}{sec}$ $PowerDen(V_{rmc}) = 4.134 \times 10^3 \dfrac{watt}{m^2}$

$Energy := \displaystyle\int_{0 \cdot \frac{m}{sec}}^{1000 \cdot \frac{m}{sec}} 0.25 \rho \cdot \left[\dfrac{k}{c} \cdot \left(\dfrac{v}{c}\right)^{k-1} \cdot e^{-\left(\frac{v}{c}\right)^k}\right] \cdot 8760 \dfrac{hr}{yr} \cdot v^3 \, dv$ $Energy = 1.811 \times 10^4 \, kW \cdot \dfrac{hr}{yr \cdot m^2}$

Figure 3. Mathcad solution for Example 1.

At the start of the worksheet, mph and kW are defined in Mathcad variables and the density (specified in the problem statement) is indicated. Functions are defined for the Weibull distribution, h(v,k,c), and the power density, PowerDen(V). The shape and scale parameters from the problem statement are inserted in the worksheet. The mode speed is 7.21 m/sec, and the power density is calculated as 230 W/m^2. The mean speed is computed using Eq. (2) to be 13.54 m/sec with a power density of 1521 W/m^2. The root-mean-cube (rmc) speed is 18.9 m/sec [using Eq. (4)], and the power density at the rmc speed is 4134 W/m^2. The use of Mathcad, and its ability to carry units in

computations, simplifies the arithmetic in this example. Since the cubic power of the wind speed is specified in the average power density expression, care should always be exercised to differentiate between the mode, mean, and rmc speeds as the power density manifests significant variation, depending which value is used.

The energy density available for collection over a year per unit area of wind turbine is specified in Eq. (3) since h(v,k,c)·8760, represents the hours per year at a given wind speed from the distribution. Figure 4 presents the energy density per year per m^2 per m/sec for the wind speed distribution specified in the problem statement. The mode for the annual energy available is near 27 m/s. Although the hours per year at 27 m/sec are small (far from the mode as illustrated in Figure 2), the product of this wind speed and the hours per year is a maximum for the distribution because of the cubic functional dependence on wind speed.

An estimate of the annual energy extracted for a Cp = 0.5 for a given wind speed distribution is provided by Eq. (8). Under the assumption of a Cp = 0.5, the annual energy density extracted for this distribution is 18,110 $\frac{kW \cdot hr}{yr \cdot m^2}$.
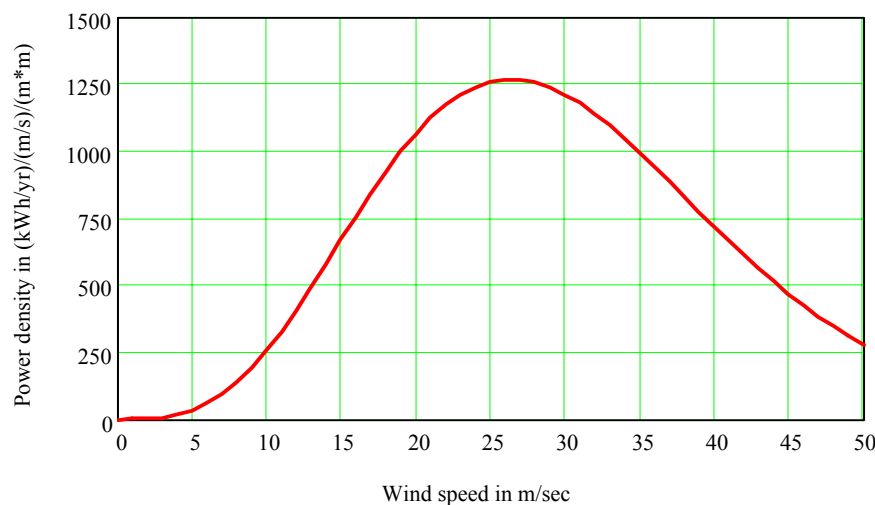

Figure 4. Annual energy available distribution for Example 4.1.

Wind Turbine Operation

What should the operating strategy be for a wind turbine as a function of wind speed? The answer is *not* to operate the wind turbine at the maximum power coefficient. This begs the question: Why not operate the wind turbine at the maximum Cp? Operation at the maximum Cp for all wind speeds would maximize the energy extracted, but factors such as generator capacity, structural requirements, and safety preclude operating at the maximum Cp for all wind speeds. The maximum speed-range will occur for only a few hours for a given wind speed distribution. So sizing a generator for an input corresponding to the maximum-speed range would result in an oversized generator that would operate at maximum output only a few hours per year. As the wind speed increased, the rotor rotation rate would have to increase to maintain a constant Cp. Since the radial stresses in a rotor are proportional to the rotation speed, operating at high wind speeds would require a structurally robust wind turbine. Additionally, as the wind speed increases safety issues and structural integrity become of increasing concern.

But, no matter what operating strategy is used, a wind turbine must contain a controller to implement the strategy and mechanical elements to respond to the controller. The ultimate purpose of a wind turbine control strategy is to regulate the power output of the turbine as a function of wind speed and direction. Additionally, the control protocol must ensure safe operation over all wind conditions. Patel [4] suggests that the power output versus wind speed characteristic of a wind turbine can be viewed as being composed of several regions. Figure 5, adapted from

Patel, illustrates a typical power output as a function of wind speed and delineates the various operating regimes and conditions. The ordinate variable is the percent of generator output. The regions and conditions are as follows:
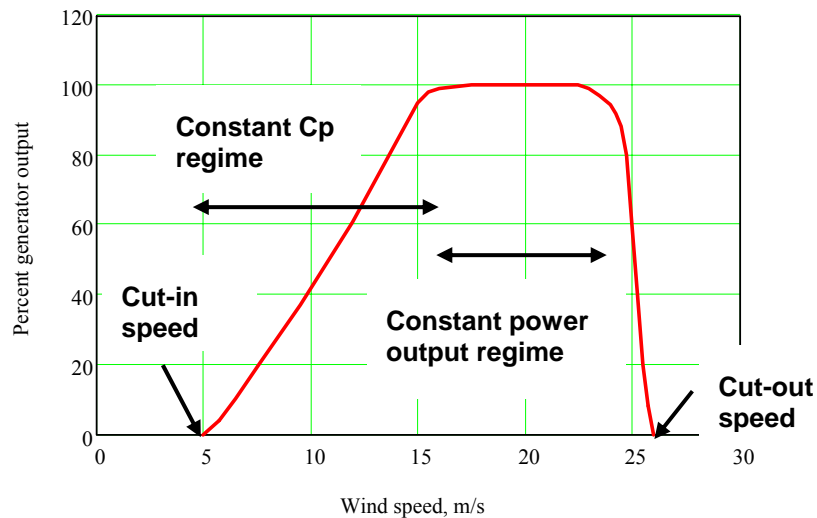

Figure 5. Typical regimes of wind turbine speed control.

The first condition is the cut-in speed of the system. Below the cut-in wind speed, the system component efficiencies are so low that running the system is not worthwhile. Once the cut-in speed is reached, the system is operated in a constant-Cp region. In the constant-Cp region, the turbine extracts the maximum power from the wind, but the power extracted is less than the rated input of the generator. When the wind speed is sufficiently high, the power extracted by the rotor can exceeds the rated input of the generator. In this regime, the constant power output regime, the system is operated to produce the rated output of the generator by operating the turbine at a Cp lower than the maximum Cp. The cut-out speed is the wind speed beyond which operation would damage the system. For speed in excess of the cut-out, the rotor pitch is set to unload the rotor and the rotor is locked with the brake. The total energy that can be extracted from a given wind distribution is reduced by the rated input of the generator and the cut-in and cut-out speeds. Example 1 will be extended to illustrate these effects.

Example 2

The system described in Example 1 is specified to have a cut-in speed of 5 m/sec, a cut-out speed of 35 m/sec, and a rate generator input of 7.5 kW/m^2. The maximum power coefficient, Cp, is 0.5. Determine and plot the following for both the system with no controls and the system controlled to meet the constraints: (a) the power density of the system, (b) the Cp versus wind speed required, (c) the energy extraction, and (d) the total energy extracted by the system.

Solution:

Much of the information requested for the "no-controls" part of this problem was accomplished in Example 1. However, since the results are more meaningful if both the no controls and the controlled versions are compared, both will be examined in the solution. Mathcad will be utilized for all the calculations. The Mathcad worksheet for the solution to this problem is given in Figure 6.

$Cp_{nom} := 0.5$ Power coefficient. $Powerin_{max} := 7.5 \cdot \dfrac{kW}{m^2}$ Generator input

$V_{cutin} := 5 \cdot \dfrac{m}{sec}$ Cut-in speed $V_{cutout} := 35 \cdot \dfrac{m}{sec}$ Cut-off speed

$PowerDen(v) := 0.5 \cdot Cp_{nom} \cdot \rho \cdot v^3$ Power density available with specified Cp.

$$PowerDenCon(v) := \begin{vmatrix} 0 \cdot \dfrac{kW}{m^2} & \text{if } v < V_{cutin} \\ Powerin_{max} & \text{if } PowerDen(v) > Powerin_{max} \\ 0 \cdot \dfrac{kW}{m^2} & \text{if } v > V_{cutout} \\ PowerDen(v) & \text{otherwise} \end{vmatrix}$$

Piece-wise continuous function defined to implement the cut-in, cut-out, and rated power constraints.

$CpV(v) := \dfrac{PowerDenCon(v) \cdot Cp_{nom}}{PowerDen(v)}$ Power coefficient as a function of speed.

$Energy(v) := PowerDen(v)\, h(v,k,c) \cdot 8760\, hr$ Energy density available with nominal Cp.

$EnergyCon(v) := PowerDenCon(v) \cdot h(v,k,c) \cdot 8760\, hr$ Energy density with controls.

$EnergyCon := \displaystyle\int_{0 \cdot \frac{m}{sec}}^{1000 \cdot \frac{m}{sec}} EnergyCon(v) \cdot \dfrac{1}{yr}\, dv$ $EnergyCon = 1.171 \times 10^4 \, kW \cdot \dfrac{hr}{yr \cdot m^2}$ Energy extracted per year per m2 for system with controls.

$Energy_{max} := \displaystyle\int_{0 \cdot \frac{m}{sec}}^{1000 \cdot \frac{m}{sec}} Energy(v) \cdot \dfrac{1}{yr}\, dv$ $Energy_{max} = 1.811 \times 10^4 \, kW \cdot \dfrac{hr}{yr \cdot m^2}$ Energy extracted per year per m2 with no controls.

$CaptureRatio := \dfrac{EnergyCon}{Energy_{max}}$ $CaptureRatio = 0.647$

Figure 6. Mathcad worksheet for Example 2.

The conditions and constraints defined in the problem statement are entered in the worksheet. The power density for the speed range specified is

$$PowerDen(v) = 0.5 \cdot Cp_{nom} \rho v^3 \tag{9}$$

The PowerDenCon(v) function, defined in Figure 6, is a piecewise continuous function in Mathcad that calculates the power density subject to the cut-in speed, the cut-out speed, and the rated input power of the generator. The first line of the function sets the power density to zero for wind speeds less than the cut-in speed; the second line constrains the power extracted not to exceed the input generator power. The third line implements the cut-out speed

constraint, and the last line ensures the constant Cp regime results. Figure 7 presents the power densities for the controlled (constrained) and uncontrolled conditions. The power coefficient, Cp, can be expressed as

$$Cp(v) = \frac{PowewrDenCon(v)}{PowerDen(v)} \cdot Cp_{nom} \tag{10}$$

and is graphically illustrated in Figure 8. The Cp is zero until the cut-in speed is reached and zero for speeds larger than the cut-out speed. The constant Cp regime is present as is the variable Cp-regime.

The power densities multiplied by the wind speed probability distribution, h(v,k,c), and the number of hours in a year, 8760, yield the energy densities. Figure 9 is a representation of the energy densities for the controlled and uncontrolled conditions. This figure is perhaps the most revealing in the solution. The effects of the cut-out speed and the generator input restriction are quite evident in the presentation. Since the areas under the curve are the total energy extracted, the effects of the control constraints are evident, especially at the higher wind speeds.
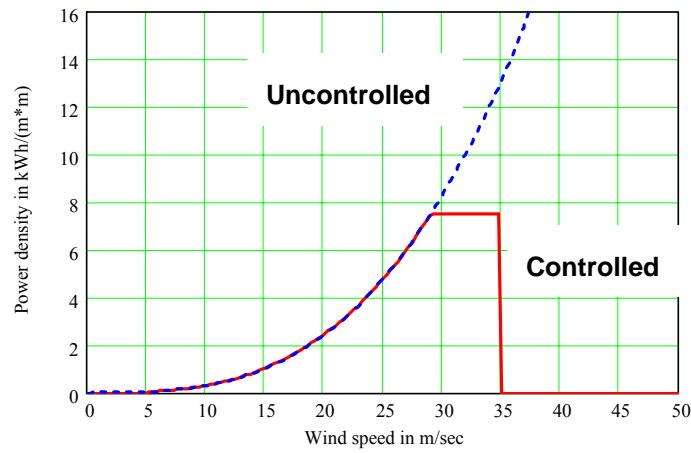

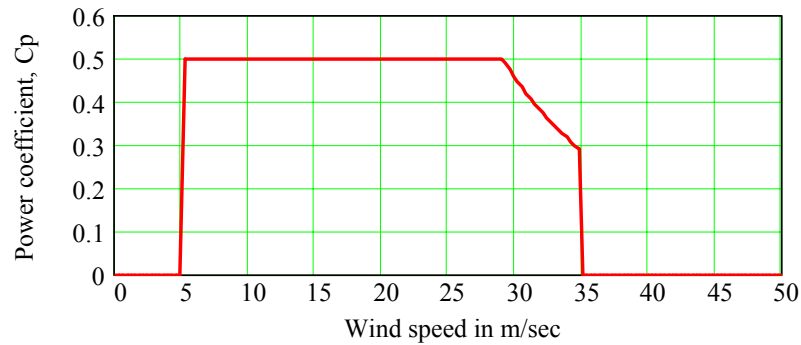Figure 7. Power densities as a function of wind speed and constraints.

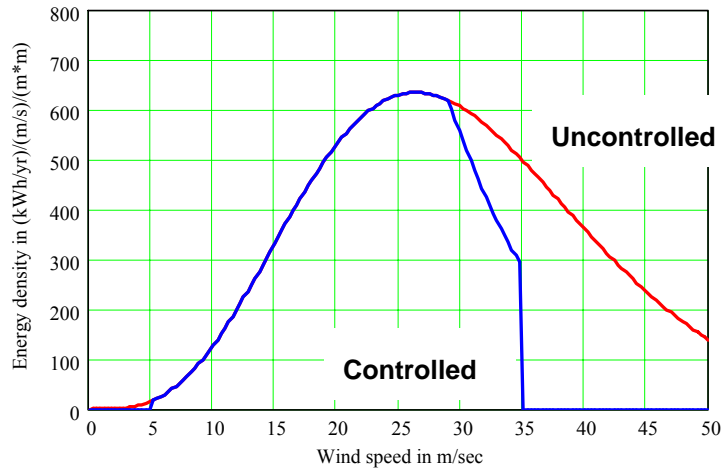Figure 8. Cp as function of wind speed.

Figure 9. Energy densities as a function of wind speed.

The energy densities integrated over all the speeds yield the total energy extracted. For the case of no control, the total energy extracted is 18,110 kW·hr/yr/m^2, (the same result as in Example 1). The actual energy extracted corresponds to the case with controls implemented. The actual energy extracted is 11,710 kW·hr/yr/m^2. The capture ratio is defined as the ratio the actual energy extracted divided by the maximum possible (no controls) for a given wind speed distribution. For the conditions of this problem, the capture ratio is 0.647; that is, 65 percent of the yearly available wind energy can be extracted.

Once the analysis of Example 2 is complete, then "what if" questions can be posed to the students. Consider how to increase the capture ratio. The only ways to significantly change the capture ratio are to increase the rated input power of the generator and to increase the cut-out speed. For the stated conditions of this example problem, neither of these strategies for increasing the capture ratio is appropriate. Increasing the rated input power would be more expensive and would mean more generator operation at less than the rated input. For example, increasing the rated input of the generator to 10.5 kW/m^2 would result in a capture ratio to 0.682, an increase of only 5.5 percent in actual energy extracted. Increasing the cut-off speed much greater than 35 m/sec would require enhanced structural integrity of the tower, nacelle, and blades.

Vestas Wind Systems, a leading manufacturer of large (MW range) wind turbines, presents an interesting demonstration of the effectiveness of their control strategy for a Vestas V-52 850 kW wind turbine. Figure 10 illustrate the response of the generator output for a V-52 wind turbine subject to variations in wind speed.

(a) Wind speed, m/s   (b) Pitch angle, degrees   (c) Generator output, kW
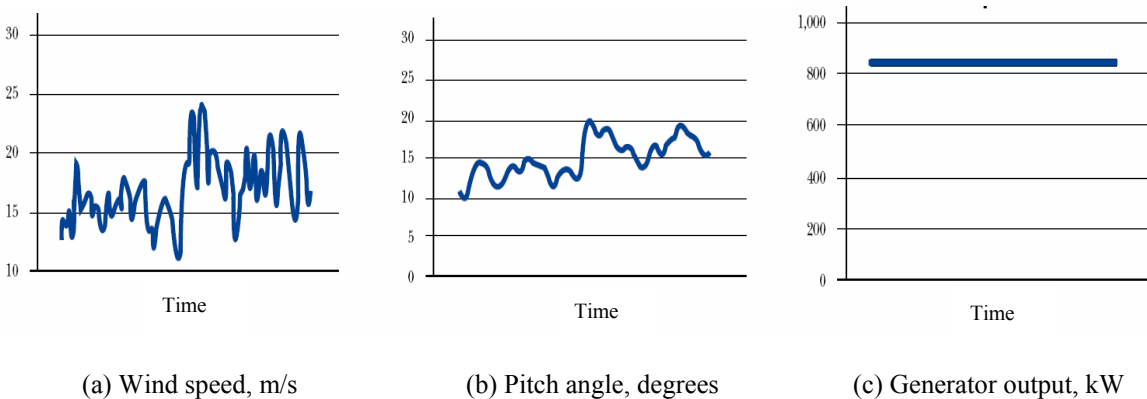
Figure 10. Vestas V-52 850 kW wind turbine generator output response to wind speed variations.

The random nature of the wind speed variation is shown in Figure 10(a). The purpose of the control protocol is to maintain the generator output at a constant value, 850 kW. In order to do this the blade pitch and the generator speed are modulated. Figure 10(b) illustrates the required modulations in pitch angle. The generator output is tracked in Figure 10(c). The success of the control protocol in maintaining a constant generator output is evident.

EXPERIENCES AND CONCLUSIONS

The use of Mathcad to enhance the quantitative coverage of wind energy systems is considered a great success. The students readily grasped that using a Mathcad-based assessment of performance metrics for a wind turbine systems was a vast improvement over just computing the power extracted for single wind speed. Moreover, the ability to pose and answer what if "questions" about wind turbine performance captured the attention of the class and permitted a homework assignment that was more focused on synthesis than analysis.

One of the reasons for the effectiveness of the use of Mathcad in providing a meaningful quantitative experience is that MSU mechanical engineering (ME) students are familiar with Mathcad and no time is spent introducing Mathcad. The MSU ME program uses Mathcad as the primary computational tool in all ME undergraduate courses. Mathcad in introduced in Thermodynamics I in the second semester of the sophomore year and forms the basis of the junior-year ME engineering analysis course. Thus, ME students are familiar with all the Mathcad syntax used in the wind energy development. About 1.5 class periods (50 minutes per period) are required to go over the examples.

In a number of conference papers and journal articles, Hodge [5 and 6, for example] and McClain [7, for example] have explored the use of computational systems, such as Mathcad, in engineering education settings. They have delineated the pedagogical advantages of such systems in engineering education and have become strong advocates for the use of computational systems in engineering education. This paper is another example of how Mathcad can be used effectively in engineering education.

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