Low Cost, Low Space, Low Setup Time Experiments for a Course in Numerical Methods

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Abstract – Five low cost, low space, low setup time experiments have been developed and implemented in an undergraduate course in Numerical Methods. Most mathematical procedures that are taught in the course are covered in the analysis and interpretation of the experimental data. This paper describes these experiments and shows how they are used throughout the course.

Keywords: numerical methods, experiments.

INTRODUCTION

One of the major and common themes during our graduating seniors exit interviews several years ago was that they would like more hands-on and more real-life applications in their mechanical engineering courses. In response to such requests, several lecture courses in our department have now incorporated experiments that include class demonstrations, collection of data in a laboratory, building of simple experiments, etc.

As part of this effort, we developed a set of five simple experiments that are now used regularly in the classroom to teach the course in Numerical Methods.

We developed experiments that

1. are low cost so that other universities can develop them with minimal material cost (some experiments need use of a university machine shop),
2. require low space so that they can be carried to the classroom or set up in the laboratory that has limited space,
3. need low set-up time so that nominal amount of classroom or laboratory time is used. This is especially important in the Numerical Methods course at USF where other educational components such as simulations, problem-centered approach, programming, and real-life project assignments are also incorporated.

Data obtained from experiments is assigned for analysis as homework or as in-class computer laboratory assignment. Comparison between experimental and numerical results is also made.

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THE EXPERIMENTS

Five experiments are incorporated in the classroom. These experiments are described below.

Experiment#1. Cooling an aluminum cylinder

In this experiment, an aluminum cylinder (Figure 1) that has two inserted thermocouples is immersed in an iced-water bath. The thermocouples placed in the cylinder are connected to a digital temperature recorder that measures the temperature vs. time data. Taking the data every ten seconds takes just a couple of minutes. The data is used for several homework exercises such as

- finding the rate of change of temperature via numerical differentiation,
- extracting coefficient of convection of iced water using regression via theoretical models.

Figure 1. Immersing aluminum cylinder in iced-water experiment

With convection coefficients depending on temperature, the theoretical model is the solution of a nonlinear ordinary differential equation that is solvable only by numerical methods. The theoretical model for the problem [Kreith, 4] is given by

$$mC\frac{d\theta}{dt} = -hA(\theta - \theta_a) \quad (1)$$

where

$h(\theta)$ =the convective coefficient, $W/(m^2\text{-}^oC)$

A = surface area, m^2

θ_a = ambient temperature of iced water, ^oC

m = mass of the aluminum cylinder, kg

C = specific heat of aluminum, $J/(kg\text{-}K)$

The ordinary differential equation is subjected to

$$\theta(0) = \theta_0 \quad (2)$$

where

θ_0 = initial temperature of aluminum cylinder, °C

In case of assuming the convection coefficient to be a constant, the exact solution to the ordinary differential equation (1) is

$$\frac{\theta - \theta_a}{\theta_0 - \theta_a} = e^{-\frac{hAt}{mC}} \tag{3}$$

The nonlinear model given in equation (3) is used as a regression model to find the average convection coefficient.

Experiment#2. Loading a Truss

A second experiment is that of an aluminum truss (Figure 2) that is loaded in the center. Strain gages are placed on three of the truss members. Balance of forces in the truss result in a set of simultaneous linear equations. The students are asked to set up these equations using the force balance method. They can then use any of the mathematical packages [Maple,5; Mathematica,6; Matlab,7; Mathcad,8] to find the force in the members on which strain gages are placed. Strain in a member can then be calculated from these forces by

$$\varepsilon = \frac{F}{AE} \tag{4}$$

where

F= force in member, *N*
A = Cross-sectional area of member, m^2
E= Young's modulus of member, *Pa*

and then compared with the strains measured by the strain gages.

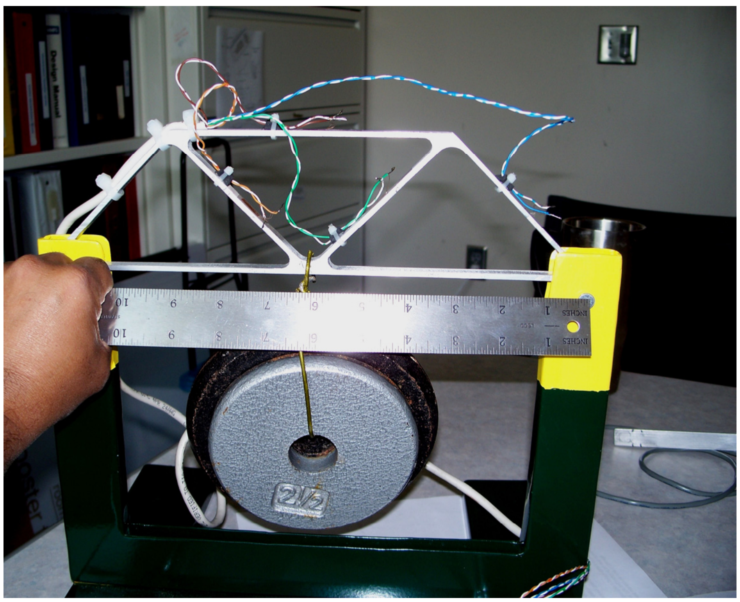

Figure 2: Loading a truss experiment

Experiment#3. Estimating the volume of a champagne glass

A third experiment takes several odd-shaped champagne glasses (Figure 3) that are measured for their outer radius at different locations along their height. Subtracting the thickness of the glass from the outer radius and using spline interpolation and integration, students estimate the volume of water these champagne glasses can hold.

Figure 3. Finding the volume capacity of a champagne glass

The spline interpolation develops the spline interpolants for the inner radius as a function of height. Then the volume of the champagne glass can be calculated as

$$V = \int_0^H \pi r^2 dh \qquad (5)$$

where

r is the varying radius of the champagne glass as a function of height, h,

H is the height of the champagne glass.

This value is then compared with the actual volume of water that the champagne glass can hold by pouring a fully filled champagne glass into a graduated cylinder.

Experiment#4. Choosing the best mousetrap

The fourth experiment is to choose the best mousetrap for the mousetrap-car contest. To do so, we want to pick the mousetrap that can store the most amount of torsional energy. We take several mousetraps and measure the force required to twist the spring as a function of angle of rotation (Figure 4). Torque T is calculated using the measured lever moment arm, that is

$$T = F\,L \qquad (6)$$

where,

F= Force applied (N)

L = Moment arm (m)

The relationship between the torque T applied and the angle θ of the rotation of the spring rotation is assumed to be a straight line

$$T = k_0 + k_1\,\theta \qquad (7)$$

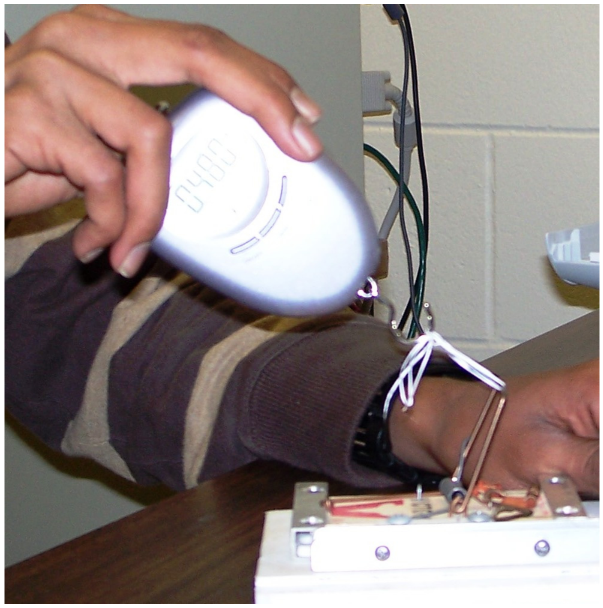

Figure 4. Twisting the mousetrap spring experiment

Using regression, the constants of the linear model in Equation (7) are found. The torsional energy stored U is then given by

$$U = \int_{\theta_{low}}^{\theta_{high}} T d\theta = \int_{\theta_{low}}^{\theta_{high}} (k_0 + k_1\ \theta) d\theta \tag{8}$$

Knowing that in our case, $\theta_{low} = 0$ and $\theta_{high} = \pi$, the maximum potential energy stored is given as

$$U_{\max} = \int_{0}^{\pi} (k_0 + k_1\ \theta) d\theta = k_0 \pi + k_1 \frac{\pi^2}{2} \tag{9}$$

This number is calculated for each of the mousetrap springs, and the one with the highest value is the one that stores the most amount of torsional energy.

Experiment#5. Finding the length of a curve

In this experiment, a flexible curve (Figure 5) of length 12" made of lead-core construction with graduations in both millimeters and inches is used to draw a curve on a graphing paper as shown. The shape of the curve is similar to the classical Runge curve of $y=1/(1+25x^2)$. This function was used by Runge [Esperson, 1] to show that higher order interpolation is a bad idea.

Once the student has drawn the 12" long curve, he/she is asked to choose several points along the curve. The student can now take the data pairs and use polynomial interpolation and spline interpolation to find the interpolated curve. One clearly notices the oscillatory behavior of the polynomial interpolant and the smooth nature of the splines. One can now find the length of the two interpolants by using

$$S = \int_{a}^{b} \sqrt{1 + \left(\frac{df}{dx}\right)^2}\, dx \tag{10}$$

where

S is the length of a path,
f is the interpolant *f(x)*,
a is the starting *x*-point, and

b is the end *x*-point.

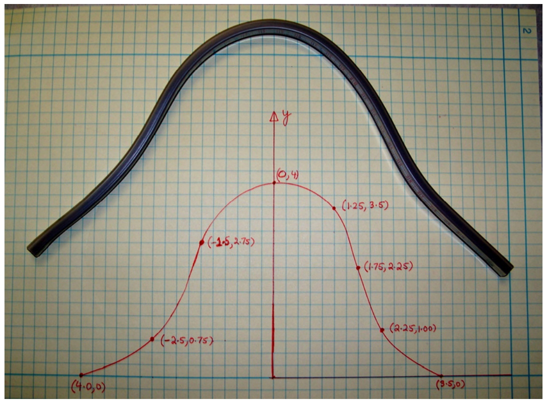

Figure 5. Using a flexible curve to draw a curve of known length

Now the length of the interpolants is compared with the actual length of the original curve drawn by the flexible curve. This exercise is also then related to a real-life problem of finding the shortest (but smoothest) path of a robot that needs to traverse through several discrete data points.

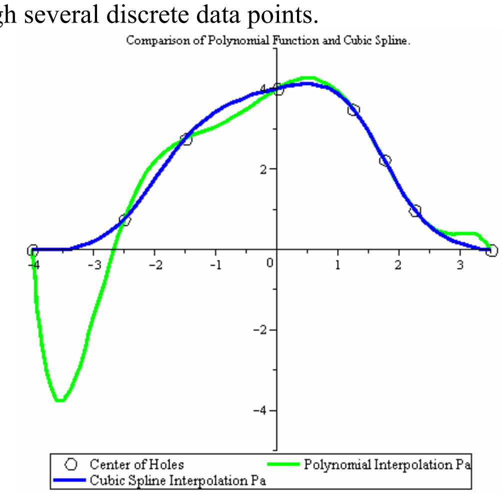

Figure 6. Comparison of Polynomial and Spline Interpolants

A NOTE TO THE READER

A quantitative and qualitative student satisfaction survey on incorporating the experiments will be conducted in Spring 2008 and Summer 2008 semesters. The results of the survey will be available first in June 2008 and then in August 2008 at http://numericalmethods.eng.usf.edu/experiments. The details of each experiment including the sketches, equipment sources, cost, and problems assigned will be provided in detail at the same website in December 2008 so that the reader can duplicate the experiment in his/her institution.

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