On Engineering Students' Algorithm Insight and Math Manipulation Abilities: A review

Gustavo J. Molina [1]

Abstract - Deficiencies in algorithm insight and mathematics manipulation abilities in college students have been the focus of recent education initiatives because they may be critical factors in students' problem-solving development. This problem has been systematically studied by Redish and Tuminaro for Physics-major students, but has not yet been investigated for other science and engineering majors. The works of Redish et al. are discussed on their relevance to engineering education: they identified six different types of difficulties with assigning meaning to the mathematics in problem context; four of them apply to engineering problem solving. Their work on identifying the most common student problem-solving schemes is reviewed. This paper also discusses the work of the author and colleagues on the disconnection between the classic math teaching with "x and y" to the use of other variable names in engineering courses. Students lack adequate training on solving actual applied-math problems in context, because they largely use "formula pattern matching" instead of relating data and unknowns to implied concepts, and because they interpret data by relating it to symbols with which they are familiar. Initiatives (i.e., curricula and class content changes) and available teaching tools to overcome such deficiencies are reviewed, and recommendations for their use are made. It is concluded that there is a substantial body of knowledge that attempts to understand students' math manipulation abilities, but experiences on application of such knowledge in the classroom are scarce. The author believes that the knowledge reviewed can help develop teaching environments and interventions to more effectively teach some engineering subjects.

Keywords: problem-solving, algorithm insight and mathematics manipulation, epistemic games.

INTRODUCTION

Deficiencies of algorithm insight and manipulation abilities in college students have been the focus of recent education research, because they can be critical factor for problem-solving development in physics and engineering. For instance, one such critical ability is symbol manipulation. Redish [Redish,1] discussed the difficulties that physics teachers observed when students are asked to apply mathematics concepts to problem solving; engineering faculty often observe those deficiencies. Students must take mathematics as prerequisites to their study of physics and engineering classes, but engineering instructors are often surprised by how little math students seem to know. It should be recognized, however, that from their successful performances in their math classes, students must have acquired the basic math concepts.

The author's interest in this educational concern started with a systematic study with colleagues at Georgia Southern University [Clark,2]. They investigated if using the classic mathematics teaching with "x and y" as variable names, rather than other names as in engineering courses, could be a significant factor in students' performance with engineering problem solving. A ten problem quiz was designed in two versions: one using only x and y, the other using a wider range of variable names. The problems were identical in all other respects, and none of the problems was calculus-based. The quiz was given to one hundred twenty-four students in first, second, and third year Engineering and Engineering Technology majors at Georgia Southern University. Students taking the xy version of the quiz scored significantly higher than students taking the mixed-variable quiz (one of the questions, however, did

[1] Department of Mechanical and Electrical Engineering Technology, Georgia Southern University, PO Box 8045, Statesboro, GA 30460-8045, gmolina@georgiasouthern.edu

not use any variable at all, and no significant difference was observed on students' performance when solving this control question). Students with higher class rankings (juniors and seniors) showed a lower difference between the quiz versions than did students classified as freshmen or sophomores. The disconnection between mathematics teaching with x and y as preferred variables and the use of more varied and descriptive names in engineering and technology courses may partially explain some students difficulties.

In a study of the knowledge and application of college algebra, Conway Link [Conway Link,3] asked students to solve for the radius of a sphere using the sphere's volume formula. He found that only 34.6% of the surveyed students were able to produce a correct answer, but that the number of mathematics courses taken after College Algebra seemed to increase the likelihood of a correct answer. He ascribed this to students' encountering an increasing number of examples and problems with "non-traditional" variables and formulas as they progress through their coursework. On discussing the problems encountered in teaching mathematics to engineering students, Sazhin [Sazhin,4] noted that changing a basic linear equation on x as variable and letter as constants, to a version on greek-letter symbols prevented most students from solving it for the unknown. He ascribed this to students' tendency to memorize equations and their manipulation in a particular notation. In general, a disconnection is seen between the student's physical understanding of the problem and the mathematical model manipulation. Klebanoff et al. [Klebanoff,5] speculated that the type of work that students are asked to perform in mathematics classes does not prepare them for applying mathematical concepts in engineering contexts. They noted the compartmentalization that exists in which students see little substantive relationship between math, science and engineering.

These experiments indicate that college students may largely use "formula pattern matching" to the employed symbols as their main method of applying math. The author and colleagues' previously referred experiment [Clark,2] tends to back this hypothesis. But other studies indicate that a deeper disconnection between doing math and using math in science may also be at the root of this problem. For instance, experienced professionals (e.g., engineers, physicists, etc) use their knowledge to see "meaning of engineering things" (or of "physics ones") in the "symbols of math" when they interpret equations [Redish,6], while student would not. This paper reviews some relevant works that help understanding deficiencies in algorithm insight and mathematics manipulation abilities in college students.

LITERATURE REVIEW ON UNDERSTANDING AND USE OF KNOWLEDGE IN APPLIED SCIENCES

As engineering instructors, we rely on curriculum design to make sure that students know the basic math concepts needed for our classes (and we can assume they do from their success in math classes). But usual engineering curricula place very little or no emphasis on students' deep understanding or use of such concepts until the first engineering classes (which are usually at the sophomore level). On investigating students' understanding of knowledge, the work of Sherin [Sherin,7] attempts answering the fundamental question: What does it mean to understand a equation? He discusses how successful students learn to understand what physics equations say in a fundamental sense; in this, he postulates that they have a feel for expressions, and that this guides their work. More specifically, students would learn to understand physics equations in terms of a vocabulary of elements that are "symbolic forms". Each of these symbolic forms, that are not just symbols but may be combinations of them, would associate a simple conceptual scheme with a pattern of symbols in an equation. Generalizing Sherin's conclusions to applied sciences and engineering, math instruction should be focus toward helping students acquire this "symbolic" understanding.

Sherin and other researchers also correctly note that college students enter science instruction with quite a lot of knowledge about the basic world, and that this knowledge may have a strong impact on their learning of more formalized (e.g., math-based) science. The study of this prior knowledge has become a research problem of its own. It is usually embodied in the so-called intuitive physics or naïve physics knowledge research. This type of research includes attempts to prove that students possess their own theories of physics [McKloskey,8], and discussions of how these theories would affect their understanding of more formalized science approaches. Clement [Clement,9,10] and McDermott [McDermott,11] extensively listed students' difficulties and misconceptions that can deeply affect the understanding of equations. It seems apparent that engineering students are not an exception to this phenomenon, but the impact of their misconceptions on learning engineering sciences has not been studied.

STUDENTS' DIFFICULTIES WITH MATH MODELING IN APPLIED SCIENCES

Extensive pedagogical research proves that students are weak at linking the mathematical world and the real world, thus supporting a view that students need much stronger experiences in building connections between real world and mathematical world [Ikeda,12, and Klymchuk,13]. The work of Crouch and Haines [Crouch,14] discusses and attempts explaining the problems faced by students in linking mathematical models to real-world applications. Their study is based on student responses to multiple-choice questionnaires on mathematical modeling problems, student reflective questionnaires, and subsequent interviews. It also investigates the processes and behaviors used by students to solve such problems. There are also initiatives that attempt to bridge this gap between math and real world in the teaching: Hiebert et al. [Hiebert,15] proposes a reform in curriculum and instruction based on allowing students to "problematize" the subject: students should be engaged in resolving problems. In mathematics, this principle has been traditionally studied under the umbrella of problem solving, while problem solving in curriculum development has been largely influenced by a distinction between acquiring knowledge and applying it. Hiebert et al, however, proposed the alternative, but presently prevalent principle of "reflective inquiry." They argue that such an approach would facilitate students' understanding, and that they will be more likely to see appropriate applications if they spend considerable time working in applied situations. Students would also acquire domain-specific knowledge while doing so. Intriguing or relevant problems will get the interests of students and engage them in mathematics.

The issue of improving student's modeling capabilities has been recently addressed by the introduction of high-school-senior and college-freshman courses which focus on problem modeling. Watson [Watson,16] shows an example of improving teaching of probability concepts in Australian schools through newspapers and the media. A typical college curriculum development implementing such pedagogy is that of a Mathematical Modeling class as an alternative for the classic College Algebra. But some problems should be consider before applying math-modeling based initiatives, as that of students' capabilities on applications and modeling tasks would likely to be influenced by the teaching approach, the context and situation in which the mathematical modeling task is embedded, student and teacher motivation, engagement with and attitudes towards modeling work, and the amount of practice and experience students had on modeling tasks [Niss,17]. Since teamwork and projects are common features of undergraduate programs in engineering and technology, it is appropriate to consider the possible effects of group-work and its organization on mathematical tasks. Research work also showed [Klymchuk,13] that while most students found application problems are more interesting, more than half of the students preferred tests that consist of pure mathematics problems because they were easier to pass, and with application problems they had difficulty moving from the word problem to mathematical language.

These changes towards math modeling teaching, however, have not been formally implemented for majors that require Calculus as basic math. That may be because there is tradition among physics and engineering faculty that Calculus teaching requires focus not only on the formulation of the mathematical model, but also strongly on abstraction and the higher-concepts that are needed for advanced-math classes. The work of Klebanoff and Winkel [Klebanoff,5] has compiled a collection of 91 complex Calculus-based problems in mathematics that are related to science and engineering; they are available at their website: http://www.rose-hulman.edu/Class/CalculusProbs. There is no formal evaluation if the use of these technology-based problems could improve student's application of their Calculus knowledge, but there seems to be a common agreement among engineering educators that student should build applicable-math acknowledge and applied-problem solving expertise. The following section of this paper discusses some aspects of the understanding and use of mathematics in applied sciences.

UNDERSTANDING AND USE OF MATH IN APPLIED SCIENCES

There are fundamental differences in the understanding and use of math, and particularly of the symbols, between the pure mathematics and applied sciences, as the engineering and the physics. Redish [Redish,1] pointed out that a big difference can be realized from the purpose of math within each science: while mathematics is interested in expressing, manipulating and deriving abstract relationships, applied sciences use math mainly to represent meaning of quantities (and not only the quantities) and its relationships for particular systems.

Redish [Redish,6] also discussed that while mathematicians have a very strictly formalized language, applied sciences (and in particular engineering and physics) use much more flexible, "context-dependent" formalisms. By comparison to pure math, formulas of applied sciences have many different kinds of constants (i.e., numbers (2, e, π), universal dimensioned constants, problem parameters, and initial condition values) and they often blur the distinction between constants and variables; they use symbols to stand for objects rather than quantities (e.g., symbols or even math-concept interpretation may be context-dependent). Redish also noted that applied sciences put the emphasis on physical meaning or math formalism for the purpose of developing a new relationship, whatever would be more useful for the goal.

From the point of view of using math to solve applied-science problems, one of the most dramatic differences is the way applied sciences put "physical-meaning" into symbols, while pure mathematics would avoid that as much as possible. And engineering and physics often use their idea of what quantity a symbol represents to decide how the math should be interpreted. In that, math-language in the sciences is context-dependent as human languages are. But there is an important practical reason for "loading" contextual meaning onto symbols, particularly in engineering: because it allows to work with complex mathematical quantities and relationships while avoiding the mathematical rigor which would be strictly required. One example clear to engineers is the treatment of concentrated forces (or simply "forces") in solid mechanics. In the typical analysis of, for instance, shear force in a simple-supported beam, the engineering approach will first develop a strict-mathematics function of shear force for portions of the beam between these concentrated forces. But we will then treat such forces very loosely as "steps in the shear-force diagram" with disregard of any more strict math treatment of such discontinuities (a mathematician would probably use a step-function, as the delta-Dirac function, to represent the concentrated force). Engineers usually "handle" complicated math issues by "switching" from math to interpretation or meaning, or even to conventions we built upon meaning. To a great extent, engineers use and switch between two parallels models, the mathematical one and the physical-contextual one, on the usually correct assumption that they can be manipulated in parallel. Another possible example of such methodology is the engineering solving of simple mechanics problems involving friction, where engineers deal with the physical change between "static friction" to "dynamics friction" coefficients by making an assumption about the outcome of the problem, that they check against the result (instead of, for instance, using the Heaviside function $\theta(hf - \Phi)$ to mathematically represent the presence of a threshold).

One of the first researchers to describe the process of modeling in such a way that it could be used for teaching applied-mathematics was Pollak [Pollak,18]. He represented the interaction between mathematics and the real world with the scheme shown in Figure 1, which is known as the "circle of modeling."

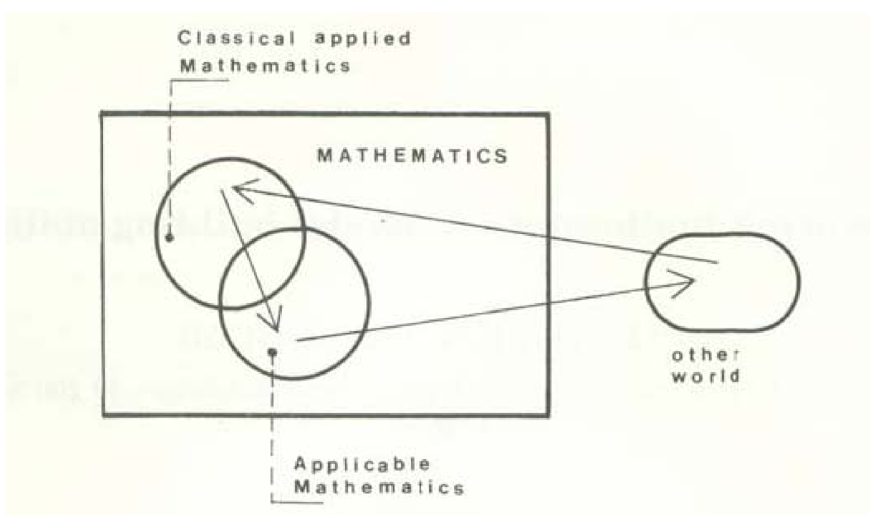


Figure 1. The "circle of modeling" (from [Pollak,18])

In this dichotomy of mathematics, according to Pollak, "classical applied mathematics" and "applicable mathematics" are two intersected but not equal sets. There are topics from classical mathematics with great theoretical interest but without any yet visible application, while at the same time there are branches of mathematics with many practical applications, but they are not characterized by many people as classical (e.g., probability and statistics, linear programming, etc). One important feature of Pollak's scheme is the direction of the arrows, suggesting a loop between the "other world" (including all the other sciences and the human activities of everyday life) and the "universe" of mathematics, and that would be the substance of what we call mathematical modeling. What engineers very often do is a "multiple looping", or a parallel development, between the applied math and the real world models. Hypotheses regarding the use of this circle of modeling are discussed by Voskoglou [Voskoglou,19]. In his paper he investigated the circle of modeling as it is employed in the classroom when the teacher gives solutions for the students and then it measures the mathematical model building abilities of them. In summary, realizing the complexity of the actual math methodologies that engineers and physicists use for solving problems can help understanding of why students seem to have deficiencies on using the math in our classes.

College Students' Typical Deficiencies when Using Symbols or Equations

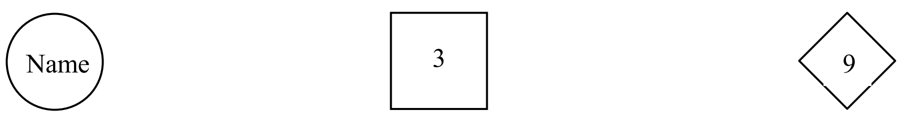
It seems that college students see little relationship between symbolic manipulations in math to the type of concept-manipulations required in engineering or physics problems. Redish [Redish,1] noted that, in a typical calculus-based physics class, the equations shown in the first week have from three to six symbols or more, and they specified a connection with something physical, while equations with a single or two symbols were typical in Calculus classes. But he also found [Redish,6] that college students often missed and did not use for problem-solving the meaning carried by used symbols or equations. He observed six different types of difficulties with assigning meaning to the mathematics in a problem context: four of these difficulties often relate to typical deficiencies encountered with engineering student problem solving, and are summarized in the following table (the two other deficiencies are "being able to parse equations" and "assigning consistent coordinate references to time and space").

Student difficulties, and deficiencies with:	**Examples where students may show such deficiencies**	**Teaching strategies and proactive actions suggested in the literature**
noting variations associated to similar symbols or equations	Not realizing the different meaning of parentheses as in: F (x,y) = k (x+ y)	Expose students earlier to variety of variable names and notations [Clark,2].
relating symbols to variables, measurements and their meaning	Students replace symbols by numbers into equations as much as possible. They do not perform dimensional analysis.	Require that students perform unit check and reality checks of the results. Ask them to interpret their results and to evaluate whether the model is adequate.
understanding equations as relationships	Understanding each equation as a "problem" in which the goal is "to find the value of x (or of the unknown)". Not recognizing equations as implicit functions between the variables.	Students should be trained in developing explicit expressions of each variable as function of the others, and in plotting them. Teach them to use structured knowledge-mapping for problem solving, as the Fuller-Polya Diagram [Molina,20,23].
treating equations as representations of reality	Students replace symbols by numbers into equations as early as possible, replacing meaning by constant values. Obtained results are unreasonable large or small.	Encourage students to formalize their problem solving strategies, mainly by including sketches/diagrams, written definition of data and unknowns, and of meaning of used symbols. Encourage students to work with symbols in the limiting cases problem (i.e., if variables become infinite, or zero). Teach heuristics approaches [Sickafus, 21] and the GENI idea [Wales,22].

Table 1. Students' difficulties with assigning meaning to math in problem context, and suggested teaching strategies

Table 1 refers to some solutions to the listed deficiencies as suggested in the literature. Some of these solutions specifically address the needs for improving student's modeling capabilities and to make them familiar to variety of variable names and notations. Redish [Redish,6] noted that students in introductory physics have a strong inclination to put numbers into their equations as soon as they know them. This makes the equations look more like the equations in their math classes and makes them seem more familiar (e.g., helping the typical "pattern matching" technique for problem solving). They also tend to drop the units; therefore, they lose the advantages of using the units as a check on errors or inappropriate mixtures of units. It seems that a primary reason that seasoned problem-solvers prefer to keep constants and data as symbols all the way to the end of a calculation, rather than putting numbers in at the beginning, is that they see equations as relationships among physical measurements. Table 1 also lists to some solutions to these deficiencies as suggested in the literature. Some of these solutions specifically address the need of improving student's modeling capabilities.

Research on problem-solving notes that good problems solvers sketch as early as possible graphs of possible relationships between involved variables and of their time evolution Molina [Molina,20,23] introduced in his sophomore mechanical-design classes a simple structured graphical method for problem solving, the Fuller-Polya diagram (FPD), that force students to see equations as relationships. FPD was outlined by Fuller [Fuller,24] from a Polya's [Polya,25] suggestion and further formalized by Kardos [Kardos, 26]. It graphically organizes the variables and their relationships in the computation, including any non-algebraic procedures (i.e., a selection among discrete values, a value read from a table or graph, etc.), while placing no emphasis on formulas or procedures details. The FPD methodology defines the following elements and standard symbols (shown in Figure 2): variable is a known or unknown "value" (it may be a number, an interval or even a code that identifies a standard part), the symbol for variable is a circle (or oval shape) enclosing the variable name; reversible algorithm is a computation or a sequence of computations that can be carried out in any "direction", even if algebra or mathematics manipulation may be needed to "reverse" such direction, the symbol for reversible algorithm is a square with an order number inside; and irreversible algorithm (or meta-operation) is a procedure that must be carried out only in a given "direction" (i.e., a double-entry table is in general an irreversible algorithm because the same output may result from different sets of inputs). The symbol for irreversible algorithm is a diamond with an order number inside, as presented in Figure 2.



Variable symbol | Reversible algorithm symbol | Irreversible algorithm symbol

Figure 2. Standard symbols for Fuller-Polya Diagram

Flowlines connect each variable symbol to every algorithm symbol, where the variable value is used in, or produced from. Figure 3 presents a simple computation sketch and FPD that was developed by a student in the author's class for multiple-leaf-spring computation; the calculation procedure can be found in standard computation handbooks.

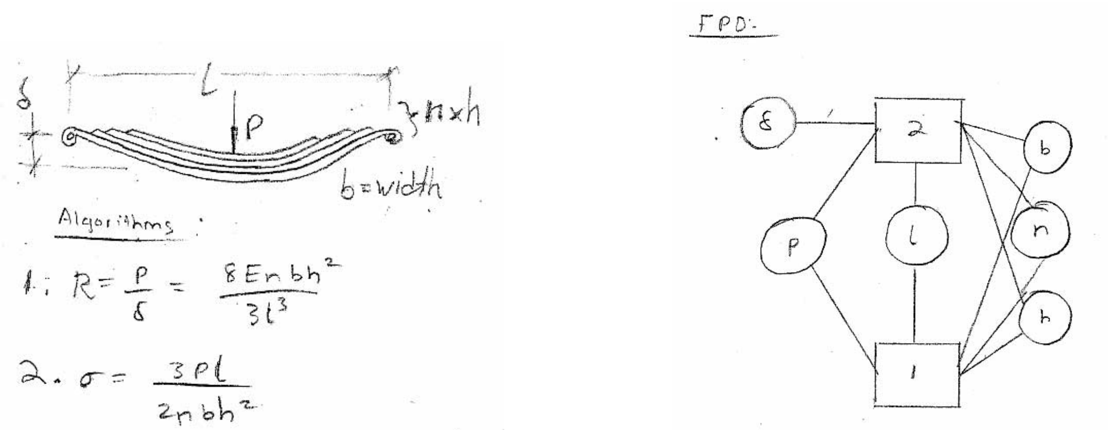


Figure 3. Sophomore engineering student work: computation sketch and FPD for multiple (n) leaf spring design.

In Figure 3, consecutive numbering for the reversible and irreversible algorithm symbols relates to corresponding algorithm descriptions that are listed on a side. Because the FPD does not need the details of formulas or procedures (but only includes them as relationships between the variables), to construct a FPD is enough to know that such relationships should exist (i.e., from physical reasoning) or that they can be established (i.e., by measurement and/or experiment). A list of variables names (and their descriptions) may be included. Several examples of FPDs were presented in the work of Kardos [Kardos,26], Molina [Molina,20,23], and Vidal [Vidal,27] . In particular, the work of Molina summarizes students' opinions (surveyed by several metrics) about the usefulness of the idea; those opinions suggested that the introduction of the FPD in a sophomore design-class helped students seeing equations as relationships between physical values in applied-math problems.

LITERATURE REVIEW ON COLLEGE STUDENTS' PROBLEM SOLVING SCHEMES

In modern educational theories, one of the dominant paradigms is constructivism, the idea that a student constructs new knowledge based largely on what that student already knows. The teacher's role in the constructivist paradigm is to create environments that help students undertake this construction accurately and effectively. But to do this, the educator needs to know the content and the structure of the students' existing knowledge. In the teaching of engineering, curriculum design makes sure that students have the knowledge contents to build upon them. But we place very little or no attention on the structure of knowledge or how they use this knowledge to construct new understanding. The various difficulties that students face when applying math knowledge to problem solving have been reviewed in previous sections of this paper. Tuminaro and Redish [Tuminaro,28] thoroughly researched the processes by which students typically attempt to solve problems (and arguably, to construct new knowledge).

The approach of Tuminaro [Tuminaro,29] proposed a useful way of analyzing students' problem solving behavior in terms of locally coherent goal-oriented activities, that they referred to as *epistemic games*. They define an *epistemic game* as: "a coherent activity that uses particular kinds of knowledge and processes associated with that knowledge to create new knowledge or solve a problem." These heuristics games both guide and limit what knowledge students think is appropriate to apply at a given time, but in general students do not choose to play these games consciously neither can articulate what game they are playing. In his PhD dissertation research work, Tuminaro [Tuminaro,30] analyzed 11 hours of video data drawn from about 60 hours of videotapes of groups of students solving homework problems in a algebra-based physics class. He described six locally coherent organizational control structures (e.g., epistemic games) that he identified students using in these problem solving activities. His work [Tuminaro,28] also includes description and examples of each of these games. They are listed below from the most intellectually complex (and most likely to succeed on developing a correct solution), to the most primitive:

(1) Mapping Meaning to Mathematics,
(2) Mapping Mathematics to Meaning,
(3) Physical Mechanism Game,
(4) Pictorial Analysis,
(5) Recursive Plug-and-Chug, and
(6) Transliteration to Mathematics.

First-four games may lead to acceptable solutions for many college-level problems. Last one, however, is an epistemic game in which students use worked examples to generate a result without developing a conceptual understanding of the examples. It is the commonly observed "pattern-matching" of formula to data, and it may lead to a correct solution if the set of data presented is as patterning as the one in the known example. The fifth game is a common (and usually unsuccessful) problem-solving scheme that leads to the so-called "plug-and-chug" cycle. This recursive scheme starts by identifying an equation that could solve the problem, to plug in it the available data; if the formula does not yields the result (or if there were no sufficient data), a new formula is sought and the cycle continues in the hope of a solution.

Tuminaro [Tuminaro,30] postulates that their decomposition of students' problem-solving in terms of epistemic games would allow instructors more clearly realize how students use their knowledge to construct new understanding, and would help to examine and guide their problem solving reasoning in more detail. He proposed that further study of these epistemic games would be useful both in understanding how to teach strategies in problem solving, and in analyzing group behavior in the context of problem solving.

CONCLUSIONS AND REMARKS

This paper presented a review on several interconnected aspects of students' understanding and use of math in applied sciences, their typical deficiencies when using symbols or equations, their difficulties with math modeling, and the typical problem-solving epistemic games they use. Some initiatives, teaching strategies and curriculum changes suggested in the literature also were reviewed. This work discussed key differences in the way applied sciences put "physical-meaning" into math, and why engineers have practical reasons for "loading" contextual meaning onto symbols. There is a substantial body of knowledge that attempts to understand engineering students' algorithm insight and math manipulation abilities. But experiences are scarce on the use of such knowledge for improving students' application of math, or for the teaching of problem solving. The author identified a few teaching strategies, proactive actions, and interventions that seem to help teaching some math-based engineering subjects. But it is apparent that application of such initiatives need to be carefully pilot in engineering curricula, and that rigorous study of learning outcomes is required to prove their effectiveness. The author particularly believes that math teaching based on early exposing students to problems of science and engineering could help them become better and more efficient problem solvers.

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Gustavo J. Molina

Dr. Molina obtained a Diploma in Mechanical and Electrical Engineering from National University of Cordoba, Argentina in 1986. Until 1992 he acquired expertise in standard testing, PVD coatings, vacuum techniques and the design of experimental equipment. In 1994 he received a Master's degree in Mechanical Engineering from the University of Ottawa, Canada, where he conducted research on the nondestructive characterization of low-energy impact properties of polymers and composites. In 2000 Dr. Molina obtained a PhD in Mechanical Engineering from Virginia Tech for his work on the characterization of electron triboemission from ceramic surfaces. Dr. Molina is currently an Associate Professor of Engineering Studies in the Department of Mechanical and Electrical Engineering Technology, Georgia Southern University. His current teaching interests include creative design, problem-solving techniques and the development of studio methods in engineering. He also teaches mechanics, graphics communication, CAD and computer applications in engineering.