Designing, Building, and Testing an Apparatus to Verify the Fourier's Law of Heat Conduction through One Semester Senior Design Course at Southern Arkansas University

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Abstract

Development of laboratory facilities in engineering programs burdened with limited financial resources is a challenging task. Faced with reduced funding from State agencies, universities strive to maintain and sustain teaching programs through the efficient utilization of in-house capabilities. This paper describes such an effort undertaken by some of our senior students to design, build and evaluate an apparatus to verify the Fourier's law of heat conduction. The team developed different concept designs, evaluated them using a weighted rated method, built it and completed test trials. Based on their evaluation, the equipment was further refined. Our test results show that it can be used to verify the Fourier's law of heat conduction successfully and measure the thermal conductivity of copper with less than 15% of error.

Key words

Design, Low-cost lab equipment, Conduction heat transfer equipment

Introduction

Verification of the Fourier's law of heat conduction in one dimension is a common heat transfer or thermal science laboratory exercise. Most undergraduate engineering programs in the United States of America (U.S.A.) and around the Globe offer this laboratory to verify the Fourier's law of heat conduction in one dimension. Even though the experiment is simple, the equipment can be expensive.

Developing laboratory facilities in an engineering program is a major challenge due to financial constraints. Many engineering programs adopt various approaches to reduce the cost associated with laboratory classes. Powell et al.[1] reported the use of web-based technology for laboratory instructions to reduce the cost. Douglas, J. and Holdhusen, M. H. [2] reported the development of hands-on lab experiments for an online course in mechanics of materials. Torick, D. and Budny, D.[3] developed a fluid mechanics lab under six thousand US dollars. Some program has adopted the approach 'Lab in a box' to minimize costs (Weitzen et al. [4]). Ahmed et al; [5] described the effort of Southern Arkansas University (SAU) to develop low cost lab equipment. These efforts are a clear sign that universities are interested in low cost lab equipment if the quality is reasonably acceptable.

The engineering program at SAU started in 2014. Development of quality laboratories with minimum cost is a major focus of its faculty. While spending money to purchase high technology equipment, any apparatus that does not need higher level of sophistication can be built in-house. Measurement of linear thermal conductivity using the Fourier's law of heat conduction is a good example for which an in-house equipment can be built. In addition, building in-house laboratory equipment provides an opportunity for students to gain valuable hands on experience in design, analysis, prototyping, and evaluation.

In this paper, an ongoing work to develop a reliable, low cost equipment to verify the Fourier's law of heat conduction in one dimension is presented. The presented work includes the testing process, experimental data, demonstration of the Fourier's law of heat conduction in one dimension, and the experimental determination of the thermal conductivity of copper.

Theory

The Fourier's law of thermal conduction in one dimension is discussed in all most all heat transfer textbooks. The textbook "Fundamentals of Heat and Mass Transfer" authored by Bergman, T.L., Lavine, A. S., Incropera, F. P., and DeWitt, D. P.[6] presents the one dimensional conduction heat transfer equation in following form.

$$q_x = -k\,A\,\frac{dT}{dx} \qquad (1)$$

where q_x is the conduction heat transfer rate in the *x* direction, k thermal conductivity, A cross sectional area of the heat transfer medium, and *dT/dx* is the temperature gradient.

The equation (1) can be rearranged as given in equation (2) to imply that the heat flux rate is directly proportional to temperature gradient. Heat flux rate divided by the temperature gradient gives the thermal conductivity. Alternatively, a plot of heat flux rate vs temperature gradient should be a straight line, from which the thermal conductivity is obtained through the slope.

$$\frac{q_x}{A} = -k\,\frac{dT}{dx} \qquad (2)$$

In linear conduction experiments, a cylindrical solid bar is used. Thermocouples are inserted in the bar at known distances. One end of the rod is exposed to higher temperature and the opposite end is exposed to cold temperature. At steady state, heat removal rate at the cold end is equal to the steady state heat transfer rate through the material. Therefore, equation (2) can be re-written in the form

$$\frac{\dot{m}\,.C_p.\theta}{A} = -k\,\frac{dT}{dx} \qquad (3)$$

where $\dot{m}$ the mass flow rate, C_p is the specific heat capacity, and θ is the temperature difference between outlet and inlet of cooling liquid. The thermal conductivity is easily obtained using equation (4) where temperature gradient is used with its sign.

$$k = \frac{\dot{m}.C_p\,.\theta}{A.\left(-\frac{dT}{dx}\right)} \qquad (4)$$

Experimental Setup and Data Collection

The experimental set up (Figure 1(a)) consists of a one-inch diameter, twelve-inch long copper cylinder with a water jacket and an inserted DC heating element. Five thermocouples (TC s), which are 1.6 inches apart, are used to measure the temperature at five different locations in the copper rod. These TC s measure the temperature at the center of the rod at each inserted location.

Thermocouple TC 8 measures the heating element temperature. Because water jacket is a cavity in the rod, the core of the rod at the cooling end is cooled enhancing axial heat conduction. The entire rod is heavily insulated. The DC heating element used in the set up is connected to a DC power transformer to control the heat flux. Temperatures of inlet water and outlet water are measured using two TC s as shown in figure 1(a). All thermocouples are connected to a PICO data logger, which is connected to a computer. PICO software is used to collect temperature data every minute. Heating element temperature measured by TC 8 remained stable throughout each trial. For example, in trial#6, the maximum and minimum temperatures of the heating element were 47.37 ^{0}C and 47.22 ^{0}C respectively. Figure 1(b) shows the actual apparatus.

Before acquiring data, the heating element voltage was set at 4.5 V and the unit was run for about an hour allowing it to reach steady state, where temperature values at all locations show steady values. Then the computer started collecting data for about twenty minutes.

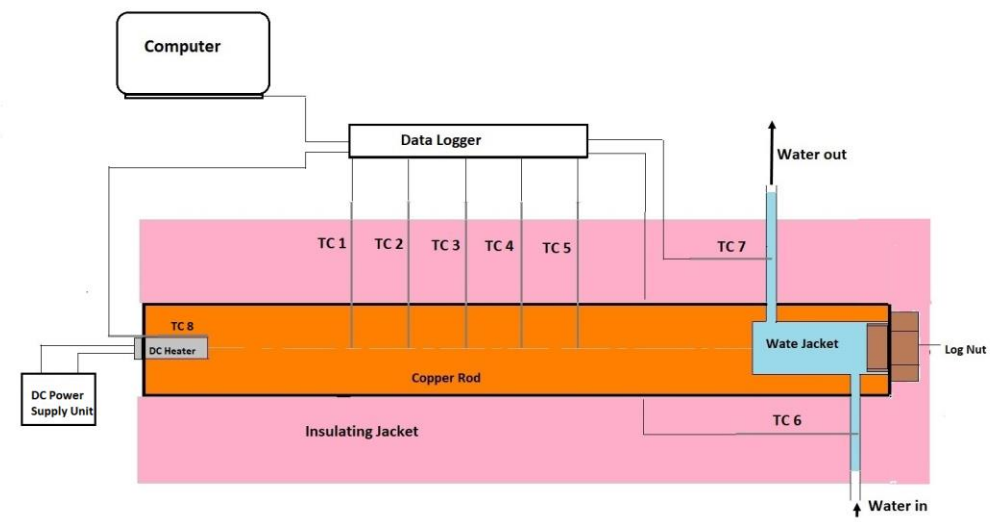

Figure1 (a): Schematic layout of the apparatus for the demonstration of the Fourier's law of heat conduction

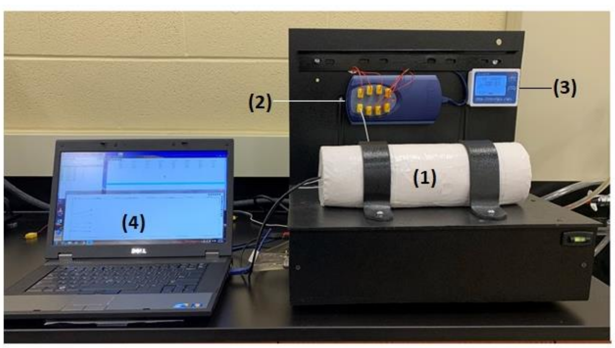

Figure 1 (b): Actual setup of the apparatus

Results

The data was collected in six trails, and a sample set of data is presented in the appendix. For each trial, steady state temperatures at measuring locations and corresponding water flow rate are presented in table 1. Table 2 provides the computed thermal conductivity of copper in each trial.

Plot of thermocouple temperatures with distance for each trail clearly demonstrates the linear variation of temperature along the copper rod. Figure 2 shows the temperature variation with the distance (x) measured from TC1 for trial#3.Thermal conductivity of copper computed for each trail using equation (3) is presented in table 2.

The Handbook of Heat Transfer edited by Rohsenow W. M., Hartnett, J. P., and Cho, Y. I. [7] and the Heat Transfer Handbook edited by Bejan, A., and Kraus, A., [8] provide a wide range of thermal conductivity values ranging from 372 W/(m . K) to 464 W/(m. K) for copper depending on temperature. The value, 372 W/(m. ^{0}C) is for industrial grade and is taken as the standard thermal conductivity value in error calculation.

Assuming that the heat flux rate is given by $\frac{(m.C\_p)^{\cdot}\ .\theta}{A}$, a plot of heat flux vs. $\frac{dT}{dx}$ can be plotted. Table 3 provides the calculated flux and the temperature gradient for trail #1, #2, #5, and #6 respectively. Trial#4 and the trial#3 have similar temperature gradient. Therefore, both trial#3 and #4 were skipped. Because the temperature gradient is negative, - $\frac{dT}{dx}$ is positive. Therefore, positive $\frac{dT}{dx}$ is used in the figure 3.

Table 1: Steady state temperature values at different locations along the Copper rod and corresponding water flow rates for different trials

Trial No.	Steady State Temperature (^{0}C)							Water Mass Flow Rate (kg/s) x 10^{-3}
	TC1	TC2	TC3	TC4	TC5	TC6 (inlet)	TC7 (outlet)	
1	25.97	25.27	24.57	23.86	23.06	22.28	22.48	4.366
2	26.27	25.52	24.76	23.99	23.20	22.30	22.65	2.603
3	25.16	24.46	23.72	22.97	22.18	21.59	21.75	5.546
4	25.35	24.65	23.91	23.15	22.37	21.57	21.83	3.501
5	25.86	25.14	24.39	23.63	22.82	22.07	22.32	3.595
6	24.20	23.28	22.66	21.92	21.07	20.22	20.41	4.761

The plot shown in figure 3 represents the variation of heat flux rate with temperature gradient for data presented in table 3. As expected, the flux rate versus temperature gradient is a straight line demonstrating the Fourier's law of heat conduction.

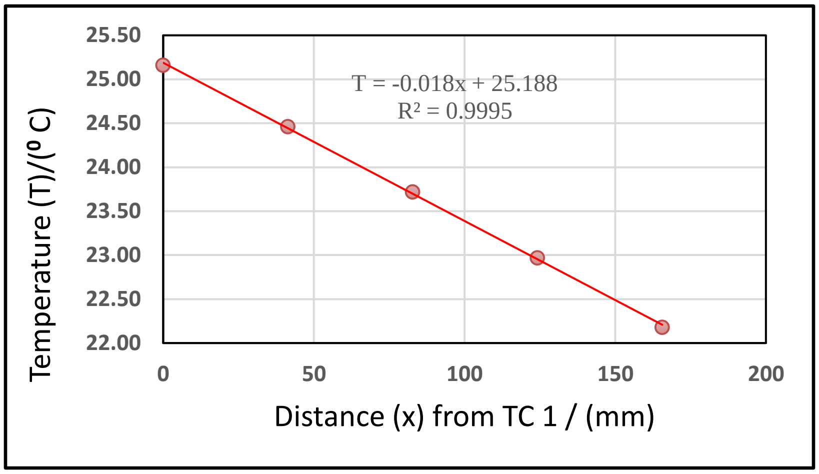

Figure 2: Variation of temperature with distance for trial#3

Discussion and Conclusion

As shown in table 2, experimentally determined thermal conductivity of copper varies between 396 W/(m.^{0}C) to 417 W/ (m.^{0}C) with corresponding percentage error varying from 6.3 to 12.2. Although it is desirable to keep the percentage error under 10%, computed percentage error is close to 10%, except in trial#4. In all trials, volume flow rate was determined by manually collecting a quantity of water during a certain time interval. However, this method can induce a considerable error in het flux rate calculation if the outlet and inlet water temperature difference is small. In the present case, the outlet and inlet water temperature difference is less than one degree of Celsius in all trials. Therefore, incorporating a flow meter that can measure small flow rates will help reduce the error.

Table 2: Computed thermal conductivity of copper

Trial No.	Thermal Conductivity of Copper (W/(m. C)	Absolute Relative True Error (%)
1	411.47	10.6
2	406.03	9.2
3	395.55	6.3
4	417.28	12.2
5	411.77	10.7
6	409.85	10.2

Table 3: Computed heat flux rates and corresponding temperatures

Heat Flux (W/m^2)	Temperature Gradient/(^{0}C/m)
7200.68	-17.57
7317.68	-18.00
7459.22	-18.40
7511.62	-18.54

The advantages of present setup are that the temperatures at different locations are taken on the centerline of the copper rod, and heating and cooling at opposite ends are directed at the core of the rod. This arrangement of heating and cooling improves the axial conduction compared to similar equipment sold by vendors.

The figure 3 represents the heat flux rate versus the temperature gradient. According to Fourier's law, the plot should show a straight line going through the origin. Figure 3 shows a straight line but is associated with an intercept, probably due to the experimental errors. The slope of the line gives the thermal conductivity of copper, which is 321.51 W/ (m.^{0}C). This value is less than the thermal conductivity values in table 2 that obtained through direct calculations. This low value of thermal conductivity obtained from figure 3 could be improved by conducting tests at different heating voltages because more accurate values for heat flux rates and temperature gradients could be obtained.

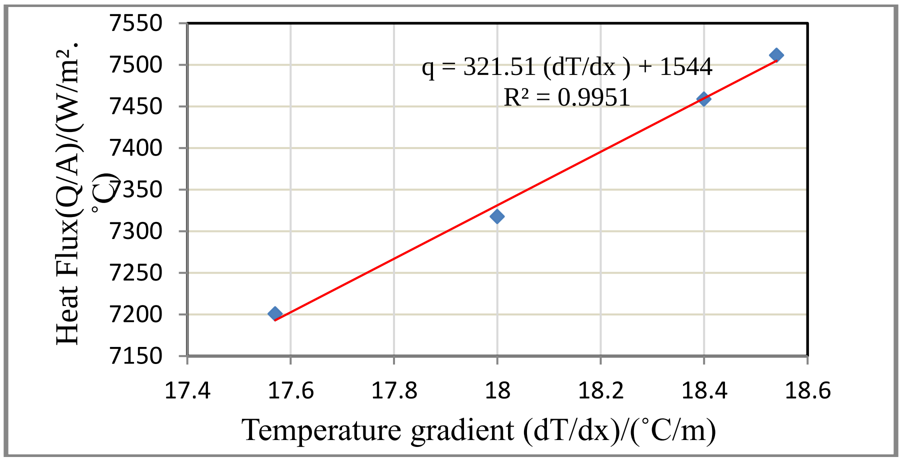

Figure 3: Variation of heat flux with temperature gradient

Table 4: Approximate cost of material and equipment

Item	Quantity	Cost ($)
1. DC Power Supply Unit	1	30.00
2. DC Heating Element	1	15.00
3. Copper Rod	1	40.00
4. Thermocouples	8	75.00
5. Insulation (3 feet length)	1	10.00
6. Flow meter with power adaptor	1	55.00
7. ABS Plastic sheet (2' x 2')	1	60.00
8. Fittings + 3/8 " plastic hose (12 ft)		45.00
9. Pico Data Logger	1	410.00
	Total($)	740.00

The total material and equipment cost was $740 approximately as shown in table 4. The computer used in the setup was a redundant one in the Engineering and Physics department of SAU and therefore not included in table 4.

In conclusion, the experimental set up presented and discussed in this paper is a viable equipment to demonstrate the Fourier's law and measure the linear thermal conductivity of copper. With some improvements in flow rate measurements, the equipment can be used in thermal science laboratory classes.

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Appendix

Sample Experimental Data - Trial#2

Time (min)	TC1	TC2	TC3	TC4	TC5	Inlet Water Temp. (^{0}C)	Outlet Water Temp. (^{0}C)
0	26.33	25.60	24.84	24.07	23.33	22.47	22.81
1	26.32	25.60	24.83	24.06	23.31	22.46	22.79
2	26.31	25.59	24.83	24.06	23.31	22.45	22.78
3	26.30	25.58	24.82	24.05	23.29	22.44	22.77
4	26.30	25.58	24.82	24.04	23.28	22.42	22.76
5	26.30	25.58	24.81	24.04	23.28	22.42	22.76
6	26.30	25.57	24.80	24.03	23.26	22.39	22.73
7	26.30	25.57	24.81	24.03	23.27	22.39	22.74
8	26.30	25.56	24.80	24.03	23.27	22.37	22.73
9	26.29	25.55	24.79	24.02	23.26	22.35	22.71
10	26.28	25.55	24.78	24.01	23.24	22.33	22.69
11	26.28	25.55	24.78	24.01	23.24	22.34	22.70
12	26.28	25.54	24.78	24.00	23.24	22.33	22.69
13	26.27	25.54	24.77	24.00	23.22	22.31	22.68
14	26.25	25.52	24.75	23.98	23.20	22.30	22.66
15	26.26	25.52	24.76	23.99	23.21	22.31	22.67
16	26.27	25.53	24.77	23.99	23.21	22.30	22.66
17	26.27	25.53	24.76	23.99	23.20	22.29	22.65
18	26.27	25.53	24.77	23.99	23.21	22.29	22.64
19	26.27	25.52	24.76	23.99	23.19	22.29	22.64
20	26.27	25.52	24.76	23.99	23.20	23.30	22.65

Computer Output of Trial#2

PLW Spreadsheet

Time Minutes	Channel 1 (°C)	Channel 2 (°C)	Channel 3 (°C)	Channel 4 (°C)	Channel 5 (°C)	Channel 6 (°C)	Channel 7 (°C)
0	26.33	25.60	24.84	24.07	23.33	22.47	22.81
1	26.32	25.60	24.83	24.06	23.31	22.46	22.79
2	26.31	25.59	24.83	24.06	23.31	22.45	22.78
3	26.30	25.58	24.82	24.05	23.29	22.44	22.77
4	26.30	25.58	24.82	24.04	23.28	22.42	22.76
5	26.30	25.58	24.81	24.04	23.28	22.40	22.74
6	26.30	25.57	24.80	24.03	23.26	22.39	22.73
7	26.30	25.57	24.81	24.03	23.27	22.39	22.74
8	26.30	25.56	24.80	24.03	23.27	22.37	22.73
9	26.29	25.55	24.79	24.02	23.26	22.35	22.71
10	26.28	25.55	24.78	24.01	23.24	22.33	22.69
11	26.28	25.55	24.78	24.01	23.24	22.34	22.70
12	26.28	25.54	24.78	24.00	23.24	22.33	22.69
13	26.27	25.54	24.77	24.00	23.22	22.31	22.68
14	26.25	25.52	24.75	23.98	23.20	22.30	22.66
15	26.26	25.52	24.76	23.99	23.21	22.31	22.67
16	26.27	25.53	24.77	23.99	23.21	22.30	22.66
17	26.27	25.53	24.76	23.99	23.20	22.29	22.65
18	26.27	25.53	24.77	23.99	23.21	22.29	22.64
19	26.27	25.52	24.76	23.99	23.19	22.29	22.64
20	26.27	25.52	24.76	23.99	23.20	22.30	22.65