Mathematics Preparation and Performance in Graduate Level Engineering Courses with Distance and Local Students

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Abstract

Preparation and performance data for this paper is derived from two graduate courses offered at the University of Alabama taught by the author. These courses include both traditional local students and distance students via video. Preparation was measured through a mathematical skills quizzes covering calculus, ordinary differential equations, and complex numbers. The skills quiz was given in the first week of class. Performance was measured through homework, quizzes, and exams. As graduate classes, the total number of students sampled is approximately 40 with multiple duplicates. Trends include a weak correlation between the initial mathematics quiz and the final course performance. The distance students appear to have a clustered distribution with exceptionally strong performance and rather weak performance. Overall, most students successfully applied calculus concepts. Most students failed to apply vector calculus and differential equations topics successfully. Mathematics skills appears stronger when the student previously took the partial differential equations (PDE) mathematics class.

Keywords

Mathematics, Distance Education, Partial Differential Equations

Introduction

Mathematics forms a critical portion of the trifecta of effective engineering: physics, mathematics, and communication. Physical understanding and modeling of processes remains fundamental. Mathematical analysis skills are necessary to convert the physics models to a system of solvable operations. Finally communication is necessary to transmit the analysis to the engineer, the boss, and the customers. As such, mathematics analysis provides the critical link between insight and solutions.

This paper focuses on the mathematics preparation and performance of University of Alabama engineers taking the author's graduate level engineering courses. The students are primarily graduate school students of both domestic and foreign backgrounds including both local and distance students. For reference, The University of Alabama operates a distance M.S aerospace engineering degree through the college of continuing studies, from which all of the distance students are enrolled. The courses including in this paper are two courses: a graduate level engineering partial differential equations (PDE) course numbered GES 554, and a graduate level airfoil and wing theory course numbered AEM 614.

Beginning in the spring of 2015, aerospace engineering department began requiring a mathematical skills quiz at the beginning of every undergraduate course. The objective was to increase the recall rate of mathematics capability among undergraduate students in response to continual complaints by professors. ABET requirements for undergraduate accreditation requires capability in mathematics and the application to engineering problems. Although the most recent ABET evaluation did not specifically mention an issue with mathematics among the AEM department's students, the professors informally and then formally identified mathematics as a potential issue. The department head formally tasked the department with forming a process to improve this issue. The recommendation approved was to create a mathematics skills exam tracking students in both subject depth and horizontally in time. The questions are required to be graded pass/fail. No partial credit is allowed to ensure a more objective evaluation. The quizzes are to be conducted in-class during an initial lecture. The skills quiz was to count as course credit (e.g. a homework grade) to ensure active participation. Each course was tasked with creating the mathematics topic coverage appropriate to the course requirements and the student prerequisites. For example, the freshman introduction to aerospace engineering course (AEM 121) only requires knowledge of trigonometry and algebra; since the prerequisites do not include calculus. In contrast, a senior level flight dynamics and control course requires significant capability in calculus, differential equations, and linear algebra.

While not strictly necessary from the department's requirements, the author began evaluating the mathematical skills among his graduate level courses starting in the spring of 2015. Prior to this evaluation, mathematics skill levels in the GES 554 course were ad-hoc and typically only resolved after assigning homework. A disconnect between the mathematics preparation expected by the professor and the preparation actually performed by the student decreased course efficiency and created multiple student-professor interactions to adjust expectations. Yet, a quantifiable set of expectations was neither fully communicated nor evaluated and characterized. An initial attempt to add more theoretical mathematics was ill received by the students; the material was simply not approachable with their background. The remainder of this paper discusses the results of this relatively short (2 semester) period of mathematics skills evaluations.

Methodology

Beginning in the spring of 2015, the author began forming a mathematical evaluation methodology for his graduate level courses. To date, two (2) courses were formally evaluated: 1) a graduate level partial differential equations (PDE) course (GES 554) open to all engineers in the engineering college, and 2) an airfoil and wing theory graduate level course (AEM 614) targeting aerospace engineering students. These courses have a different structure and different student distributions as will be quantified later.

The PDE course is a 3 hour class with the objectives of studying the theory, classification, formulation, relevancy, analysis and solutions of PDEs in engineering environments. Both analytical and computational methods are presented and evaluated. Topics include PDE classification and canonical forms, parabolic and diffusion equations, Laplace and Fourier methods, elliptic boundary value problems (BVP) equations, Green's functions, hyberbolic equations with linear and non-linear conservation equations, systems of equations, numerical solution techniques, error analysis, Monte Carlo methods, calculus of variations, perturbation, and conformal mapping methods. The class covers a significant amount of material and requires (and expects) capability in calculus, ordinary differential equations, linear algebra, and computer programming skills. The course uses Farlow's PDEs for Scientists and Engineers book[1] as a topic guide and homework problem source. Advanced topics are introduced from Salsa's more mathematically heavy textbook Partial Differential Equations in Action[2]. Evaluations are conducted through semi-weekly homework quizzes, monthly projects, and monthly exams.

The airfoil and wing theory course (AEM 614) is a 3 hour class with the objectives of investigating the aerodynamics of airfoils and wings. The course includes both physics, engineering, and flight test perspectives. Topics include fundamental physics responsible for aerodynamics, Joukowski transformations, behavior and characteristics of subsonic, supersonic and transonic surfaces, the development of panel method solvers, the analysis of airfoils and wings with commercial analysis tools, and the theory and analysis of wind-tunnel experiments. This course requires the use of calculus, ordinary differential equations, and PDE solutions for both subsonic parabolic and supersonic hyperbolic field equations. The course uses Flight Vehicle Aerodynamics[3] and Aerodynamic Design of Transport Aircraft[4] as textbooks. Derivation and use of the Prandtl-Glauert governing equations for compressible flow exercise the applied solutions of engineering problems.

$$(1-M_\infty^2)\phi_{xx}+\phi_{yy}+\phi_{zz}=0$$

In particular, the PG equation above changes behavior as the Mach number passes through M=1. Students are expected to transform compressible flow geometries to incompressible approximations through the use of coordinate transform mapping. Course evaluations are conducted through projects and exams.

The initial PDE skills exam conducted in the spring of 2015 is presented in Figure 1. The quiz is composed of 7 questions ranging in topics from calculus to ordinary differential equations and complex numbers.

The calculus portion includes the use of the chain rule,

$$\frac{d}{dx}\left(A\cdot B\right)=\frac{dA}{dx}B+A\frac{dB}{dx}$$

the fundamental theorem of calculus,

$$F(a)-F(b)=\int_a^b f(x)$$

and the orthogonality of harmonic functions

$$\int_0^1 \sin(n\pi t)\sin(\mathrm{m}\,\pi t)\,\mathrm{dt}=\begin{cases}1/2 & if \quad n=m\\ 0 & otherwise\end{cases}$$

GES 554: Math Skills Quiz Name:______________________

- Calculus: [15 pts each]
  1. $\frac{d}{dx}\left(e^x x^2\right)$
  2. $\frac{d}{dx}\int_0^x f(x)\,dx$ where f(x) is well behaved
  3. $\int_0^1 \sin(n\pi t)\sin(n2\pi t)\,dt$ where n is a positive integer
- Ordinary Differential Equations: [15 pts each]
  4. $y' - y = 0 \qquad y(0) = 1$
  5. $y'' + y = 0 \qquad y(0) = 0 \quad y(\pi/2) = 1$
  6. $\frac{dy}{dx} - y = e^x \qquad y(0) = 1$
- Complex Numbers [10 pts]
  7. Given $\left\{\begin{array}{l} u = 1 + i \\ g(\mathrm{z}) = \arg(\mathrm{z}) = \angle \mathrm{z} \\ \mathrm{f}(\mathrm{z}) = z^2 \end{array}\right\}$, Find $g(f(u))$ in radians.

Figure 1 Partial Differential Equations Skills Quiz, Spring 2015

These particular skills are used continuously during the course. In particular the orthogonality of harmonic functions is a critical identity when working with Fourier expansions of modeshapes. The ODE portion included a simple 1st order linear homogeneous equation

$$y' - y = 0 \text{ with } y(0) = 1$$

The solution is the exponential function

$$y(x) = e^x$$

The next ODE is a 2nd order linear equation for evaluating the student's knowledge of harmonic sine and cosine equations. The final ODE problem is a more involved non-homogeneous 1st order ODE. The homogeneous solution is an exponential function, thus the expected non-homogeneous term involves the homogeneous part and a particular part

$$y(x) = (x+1)e^x$$

This problem is meant to test the understanding of particular solutions and the relationship to homogeneous solutions. A range of ODE problem difficulty was specifically designed into the skills exam to provide the maximum testing range within the limited time period. A firm understanding of ODE solutions is critically necessary for PDE solutions, as the classical solution techniques often decompose the PDE to separated ODE problems. The complex number portion tests the concepts of complex number operations with a multiplication and angle operation. Complex numbers are used in the course for conformal mapping of Laplacian boundary value problem solutions to more complex domains.

The following semester, fall 2015, a mathematics skills quiz was given to students in the airfoil and wing theory class (AEM 614). The quiz is presented in Figure 2. The quiz included 4 mathematics problems, 2 fluid dynamics problems, and an aircraft identification problem. The four mathematics problems represent calculus, ODE, vector calculus, and complex numbers operations. The calculus problem is a chain rule problem. The ODE problem is a non-homogeneous 1st order equation simplified from the previous semester's non-homogeneous problem. Since aerodynamics frequently uses vector calculus, a question asking students to compute several operations was included for problem #3. The solution is zero. This problem in particular is relatively long and tedious without the identification of a vector calculus identity that the divergence of curl is zero. Thus, this is either a one-line problem or a one-page problem. To close up the mathematics skills portion, a complex numbers problem similar to the spring 2015 problem was included. In total, these skills are representative of the mathematics required for developing aerodynamics models from physics.

Students were given the exams on the 1st Friday lecture of the semester. The quizzes were fifteen (15) minutes long and were closed book, closed notes, and no calculator. Student performance was tracked and recorded for individual problems. Students, when applicable, were tracked across courses.

AEM 614: Aerospace Engineering Skills Quiz Name:___________________________

- Calculus: [10 pts]
  1. $\frac{d}{dx}\left(e^{x}x^{3}\right)$
- Ordinary Differential Equations: [10 pts]
  2. $y' - y = 1 \qquad y(0) = 1$
- Vector Calculus: [10 pts]
  3. Given $V = \left(\sin(yz) \quad e^{xyz} \quad (xyz)^{3}\right)$, compute $\nabla \bullet (\nabla \times V)$
- Complex Numbers [10 pts]
  4. Given $\left\{\begin{matrix} u = 1 + i \\ g(\mathrm{z}) = \arg(\mathrm{z}) = \angle \mathrm{z} \\ \mathrm{f}(\mathrm{z}) = z^{4} \end{matrix}\right\}$, Find $g(f(u))$ in degrees.
- Fluid Dynamics [10 pts each]
  5. Define *Reynolds number*

  6. Define *dynamic pressure*

- Identify indicated aircraft components. Be as specific as your knowledge allows (e.g. DC-3 starboard Frise aileron) [10 pts each]

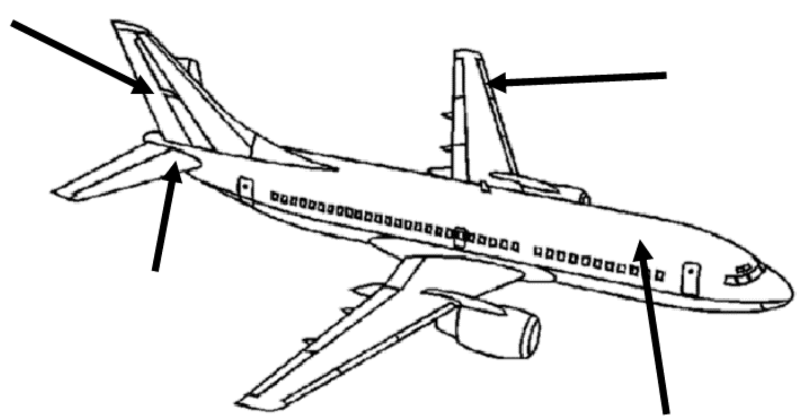

Figure 2 Skills Quiz for AEM 614, Fall 2015

Results and Analysis

The GES 554 results from the spring of 2015 are given in Figure 3. A total of 21 students took the exam, including 3 distance students. The top score was 100%; the low score was 15%.The average was 54%. Of the 6 students earning a 40% or less, only 1 completed the course. The sole survivor earned a B. Distance students earned well below average and eventually dropped the course. A comparison with the previous year (spring 2014) indicates that distance students tend to either be well prepared (#1 and #4 in class of 16) or poorly prepared (#14 in class of 16).

GES 554 Partial Differential Equations:
Math Skills Quiz (15 minutes)

Rank	Score	Product Rule #1 (15pts)	Fund' Theo' Calc' #2 (15pts)	Orth' Integral #3 (15pts)	ODE 1st #4 (15pts)	ODE 2nd #5 (15pts)	ODE 1st (NH) #6 (15pts)	Complex # #7 (10pts)	AEM UG?	Distance
1	100	● 1	● 1	● 1	● 1	● 1	● 1	● 1		
2	90	● 1	● 1	● 1	● 1	● 1	● 1	○ 0		
3	85	● 1	● 1	● 1	○ 0	● 1	● 1	● 1		
4	85	● 1	● 1	● 1	● 1	○ 0	● 1	● 1		
5	70	● 1	● 1	○ 0	● 1	● 1	○ 0	● 1		
6	60	● 1	○ 0	● 1	● 1	● 1	○ 0	○ 0		
7	60	● 1	● 1	● 1	● 1	○ 0	○ 0	○ 0		
8	60	● 1	● 1	● 1	● 1	○ 0	○ 0	○ 0		
9	60	● 1	● 1	○ 0	● 1	● 1	○ 0	○ 0		
10	55	● 1	○ 0	● 1	● 1	○ 0	○ 0	● 1	⚑ 1	
11	55	● 1	○ 0	● 1	● 1	○ 0	○ 0	● 1		
12	55	● 1	○ 0	● 1	● 1	○ 0	○ 0	● 1		
13	45	● 1	○ 0	● 1	● 1	○ 0	○ 0	○ 0		
14	45	● 1	○ 0	○ 0	● 1	● 1	○ 0	○ 0	⚑ 1	
15	45	○ 0	● 1	○ 0	● 1	● 1	○ 0	○ 0		★ 1
16	40	● 1	○ 0	● 1	○ 0	○ 0	○ 0	● 1	⚑ 1	
17	30	● 1	○ 0	● 1	○ 0	○ 0	○ 0	○ 0		★ 1
18	30	● 1	● 1	○ 0	○ 0	○ 0	○ 0	○ 0		
19	30	○ 0	○ 0	● 1	● 1	○ 0	○ 0	○ 0		
20	30	● 1	● 1	○ 0	○ 0	○ 0	○ 0	○ 0		★ 1
21	15	● 1	○ 0	○ 0	○ 0	○ 0	○ 0	○ 0		
	Aggregate	90%	52%	67%	71%	38%	19%	38%		

Figure 3 Quiz Results for Spring 2015

90% of students successfully solved the chain rule problem. Approximately half solved the fundamental theorem of calculus problem. Approximately 2/3 solved the orthogonal integral. 71% solve the ODE problem; however, the non-homogeneous 1st order problem had only a 19% solution rate. The 2nd order ODE and complex number problem both had a 38% solution rate. These scores indicate that students entering graduate school are weak at ODE solutions.

From Figure 4, the overall performance versus math quiz score is relatively weak (slope of 0.0027) when comparing the students who finished the class. When comparing the performance of all 21 students, the slope is a more significant 0.036. These results suggest that the incoming mathematical capability is a poorly correlated to class performance. However, students who performed relatively poorly are likely to drop the course.

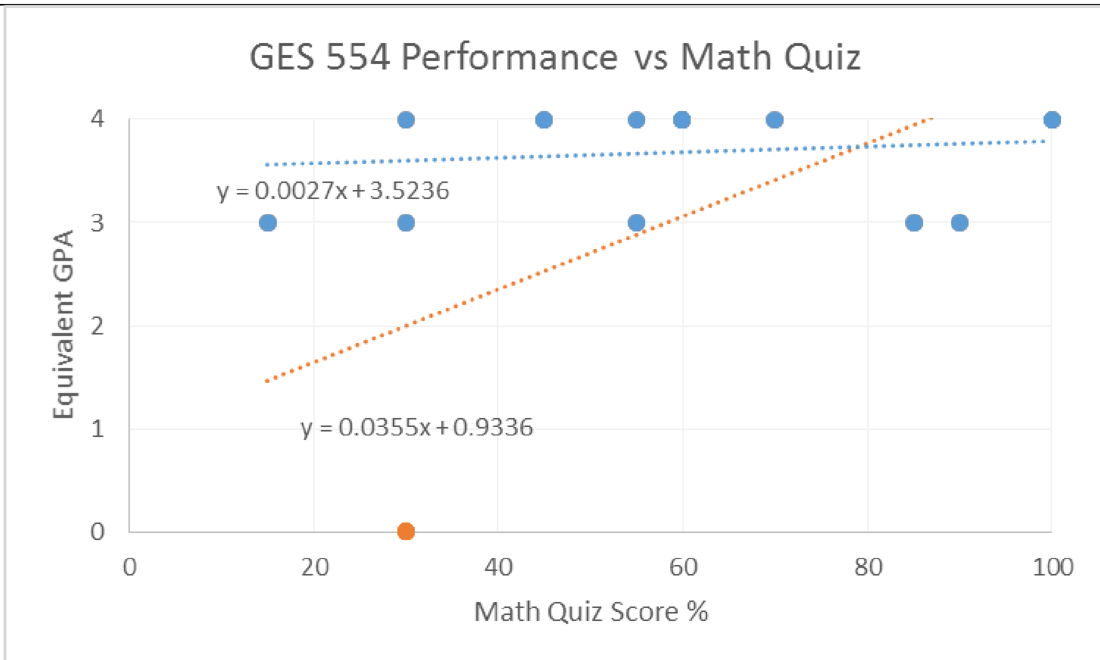

Figure 4 GES 554 Performance vs Math Quiz Score

The AEM 614 results from the fall of 2015 are given in Figure 5. At total of 20 students took the exam, including 7 distance students. The high score among the math only questions was 60%. The low score was 20%. The average was 32%. Among only the distance students, the average was 34%. Local students with University of Alabama BSAE degrees had a 35% average.

The AEM 614 quiz results reflect the previous GES 554 results. The chain rule received 100% success. The non-homogeneous ODE was solved by only 30% of students. Only 2 students, 10%, solved the vector calculus question. 20% solved the complex numbers question.

Lateral tracking is available for 6 students who previously took the GES 554 course. Their average performance is 40%. From a prior instructor viewpoint, this performance is particularly troubling even if the absolute performance is on average 8% better (or 25% better relatively). However, 3 of the top 4 students in the AEM 614 course previously took the GES 554 course.

Rank	AEM 614 Skills Quiz	Chain Rule	NH ODE	Div Curl	Numbers	Reynolds	Dynamic	Rudder	Horizontal	Slat	Fuselage	AEM UG?	Distance?
1	80%	● 1	● 1	○ 0	● 1	○ 0	● 1	● 1	● 1	● 1	● 1		☆
2	80%	● 1	● 1	○ 0	○ 0	● 1	● 1	● 1	● 1	● 1	● 1		☆
3	80%	● 1	○ 0	○ 0	● 1	● 1	● 1	● 1	● 1	● 1	● 1	⚑	
4	80%	● 1	● 1	○ 0	○ 0	● 1	● 1	● 1	● 1	● 1	● 1	⚑	
5	80%	● 1	● 1	○ 0	● 1	● 1	○ 0	● 1	● 1	● 1	● 1	⚑	
6	70%	● 1	○ 0	○ 0	○ 0	● 1	● 1	● 1	● 1	● 1	● 1		☆
7	70%	● 1	○ 0	○ 0	● 1	○ 0	● 1	● 1	● 1	● 1	● 1		☆
8	70%	● 1	○ 0	○ 0	○ 0	● 1	● 1	● 1	● 1	● 1	● 1	⚑	
9	70%	● 1	○ 0	● 1	○ 0	○ 0	● 1	● 1	● 1	● 1	● 1	⚑	
10	60%	● 1	● 1	○ 0	○ 0	○ 0	○ 0	● 1	● 1	● 1	● 1		☆
11	60%	● 1	○ 0	○ 0	○ 0	● 1	○ 0	● 1	● 1	● 1	● 1		☆
12	60%	● 1	○ 0	○ 0	○ 0	● 1	○ 0	● 1	● 1	● 1	● 1		
13	60%	● 1	○ 0	○ 0	○ 0	○ 0	● 1	● 1	● 1	● 1	● 1		
14	60%	● 1	○ 0	○ 0	○ 0	○ 0	● 1	● 1	● 1	● 1	● 1	⚑	
15	50%	● 1	○ 0	○ 0	○ 0	○ 0	○ 0	● 1	● 1	● 1	● 1		☆
16	50%	● 1	○ 0	● 1	○ 0	○ 0	○ 0	● 1	○ 0	● 1	● 1	⚑	
17	50%	● 1	○ 0	○ 0	○ 0	○ 0	○ 0	● 1	● 1	● 1	● 1	⚑	
18	40%	● 1	● 1	○ 0	○ 0	○ 0	○ 0	○ 0	○ 0	● 1	● 1		
19	40%	● 1	○ 0	○ 0	○ 0	○ 0	○ 0	● 1	● 1	○ 0	● 1		
20	30%	● 1	○ 0	○ 0	○ 0	○ 0	○ 0	● 1	○ 0	○ 0	● 1	⚑	
	Aggregate	100%	30%	10%	20%	40%	50%	95%	85%	90%	100%		

Figure 5 Skills Quiz, Fall 2015

Distance students are uniformly distributed among the quiz scores when only considering the mathematics. The distance students are relatively stronger at fluid dynamics and aircraft identification. These results were contrary to the author's informal analysis and show the need for actual measurements and systematic tracking.

The correlation between the skill quiz and the course grade for AEM 614 is near zero. The skill quiz shows no particular capability to evaluate the final performance of students. However, an unmeasurable benefit was noticed. Students voiced a concern with the poor mathematics skills demonstrated in the skills quiz. The value of the skills quiz may lie more as a catalyst to future action rather than an evaluation.

Conclusion

A mathematics skills quiz of students taking two graduate level courses over two semesters indicates weak to zero correlation between the skills quiz score and the course performance. The quizzes strongly suggest that incoming graduate students have relatively weak mathematical skills especially in the areas of differential equations and complex numbers. Among the worst performance for both semesters was for nonhomogeneous ODEs (both semesters) and vector calculus (one semester). A previous graduate level PDE math course appears to slightly improve scores by about half a question. A math skills difference between local and distance students did not appear. A larger sample size is needed for higher fidelity analysis. The author recommends a mathematics skills quiz for the measurement of student preparation.

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