Creating Engineering Teaching Modules for High School Math Courses

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Abstract – Incorporating engineering examples in K-12 math courses is a highly beneficial and desirable teaching practice. Unlike science courses, math subjects require more effort and additional knowledge to be related to practical applications. In this paper, we describe our recent effort to create two engineering teaching modules for high school math courses. The first module introduces the modeling of Direct Current (DC) circuits using linear systems of equations, while the second one introduces the analysis of signal processing in a simple wireless communication receiver using trigonometric identities. The modules not only inspire students' interests in learning math subjects, but they also have the potential to motivate some students to choose engineering as their major in college. We believe equipping K-12 teachers with basic engineering knowledge is critical, and engineering teaching modules should be widely disseminated among K-12 teachers.

Keywords: K-12 Outreach, Engineering Teaching Modules

INTRODUCTION

It is well known that America's security, economic well being, and ability to maintain a competitive edge largely depend on a scientifically literate and competent generation of young people. Due to the increasingly evident challenges in post-secondary Science, Technology, Engineering, and Mathematics (STEM) education, more and more university faculty members are realizing that K-12 education is critical to the preparation of students for STEM studies in college [1-3]. In particular, K-12 math curriculum lays the most important foundation for STEM studies [4].

To better meet the nation's critical educational needs in STEM disciplines, new methodologies and learning environments must be created to inspire students' interests in these subjects. Incorporating practical engineering examples in the classroom is a highly beneficial and desirable teaching practice, providing students with direct exposure to real world problems and sometimes the experience of new discoveries [5, 6]. Studies have shown that students with such exposure have improved academic performance and lower attrition rates. This is especially true for women and underrepresented groups [7]. Based on our teaching experiences at both high school and college levels, it is always evident that students are best motivated when they are engaged in real world problems. Involvement of students in practical engineering problems also nurtures the development of critical and creative thinking skills.

However, there are significant difficulties to incorporate practical engineering examples at the pre-college level, especially for K-12 math classes. Unlike science courses, math subjects require more effort and additional knowledge to be related to practical applications. Currently, many math teachers use examples in biology, medicine, economics or business in their classrooms, but very few engineering examples are being introduced to students.

In this paper, we describe our recent effort in creating engineering teaching modules for high school math courses. Our project was part of a six-week summer program at Embry-Riddle Aeronautical University (ERAU), and was funded by a National Science Foundation (NSF)'s Research Experience for Teachers (RET) grant. The program pairs up a university engineering faculty member and a K-12 math or science teacher, and provides the teacher an

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opportunity to participate in engineering research under the supervision of the faculty member. An important goal of ERAU's RET program is to create teaching modules describing engineering examples related to K-12 curriculum, and let the teachers bring them back to their classrooms and school districts.

BACKGROUND OF THE RET PROJECT

The RET program is an NSF program that allows K-12 teachers from across the country to work under university professors doing research in different fields of engineering. The RET program at ERAU focuses on research applications in engineering, concentrating on the fields of aviation and aerospace. Teachers accepted into this six-week program strive to incorporate what they learn at ERAU into their respective teaching curriculum in order to encourage their students to pursue studies in the fields of engineering.

The RET program at ERAU brings in ten K-12 teachers per year to participate in this learning experience for the summers of 2010, 2011, and 2012. It is funded by the American Recovery and Reinvestment Act of 2009. Each visiting teacher is mentored by a university professor on campus grounds in different research fields during their stay. The 2010 summer program is finished in late July, and final curriculums and lesson plans are also made available on the program's website: **http://www.ret-erau.com/**.

TEACHING MODULES WE DEVELOPED

Over the six-week period in Summer 2010, we developed two electrical engineering modules for high school math courses. The first module introduces the modeling of Direct Current (DC) circuits using linear systems of equations, while the second one introduces the analysis of signal processing in a simple wireless communication receiver using trigonometric identities. The teaching modules can be distributed to students as handouts, and teachers will go over the materials in the handouts in class. Students may be required to read the handouts before coming to the class, and an assignment is given at the end of each module after its content has been covered in class.

In the development process, we paid special attention to the explanation of engineering concepts in a simplified way appropriate for high school students. It is extremely important to be cautious in this area, because confusion and failure to understand relevant engineering concepts may frustrate students, and be detrimental to the cultivation of their interests.

Module 1: Using Systems of Linear Equations to Solve Circuit Problems

In this module, we introduce the mathematical model of linear electric circuits, i.e., linear systems of equations. High school students study how to solve linear systems of equations in algebra courses, and they need to be motivated to put in much effort to practice this important skill. Proficiency in solving linear systems of equations is critical in many engineering disciplines.

a. Introduction

The first part of the teaching module is an introduction that seeks to capture students' attention. Here is an example of two paragraphs that can be used:

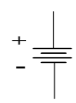
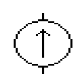
"Ever wonder how your cell phone is able to pick up your friends voice and send your voice to your friend? How you can only hear his/her voice? How you can send text messages? Browse the web by simply pressing a few buttons?

"The modern electronic devices are built with electric circuits. Circuit analysis is a first step in studying the essentials of Electrical Engineering, the engineering discipline that designs and develops modern electronic devices. Before you start to study electric circuit analysis, you need some essential math concepts and skills."

b. Basic Concepts

This section contains fundamental concepts of circuit analysis. Depending on how much students have been exposed to electricity-related concepts in a physics class, the materials can be customized. In general, the section will contain concepts of current, voltage, resistor, voltage source, ground, node, etc. The concepts are best presented in a table. For each concept, the definition, symbol, unit, and measurement tool are described in the table. Below is the table we developed.

Table 1. Basic Circuit Concepts

	Definition	Units	Symbols	Measurement Tool
Current (I)	Charge in motion	Amps (A)		Ammeter
Voltage (V)	Total work per unit charge required for moving a charge between two points	Volts (V)		Voltmeter
Resistor (R)	An electrical device that resists the flow of electrical current (i.e. light bulb, speaker)	Ohms (Ω)		Ohmmeter
Voltage Source (V_S)	Power source that supplies a pre-specified voltage. (i.e. battery)			
Ground	The reference point in the circuit considered to have zero voltage potential. It can be chassis ground or earth ground.			
Current Source	Power source that provides a circuit with a pre-specified current. (i.e. battery charger)			
Node	A junction where two or more circuit branches join together.			

c. Fundamental Circuit Principles

The section contains descriptions of Ohm's law, resistors combinations (series and parallel), Kirchhoff's voltage law and current law. For each principle, an example resistor circuit is shown together with corresponding voltage/current equations for illustration. For example, **Figure 1** is the circuit used to describe Ohm's law.

Following the example circuit, a "Puzzle" section is designed to reinforce the circuit concept. The section contains simple exercise problem for students to finish either in class or as an assignment. For example, to help students understand the linear relationship between a resistor's voltage and current, **Table 2** is given, which lists different current values (I) across a resistor and corresponding voltage values (V). Then, the students are asked to plot these points on a VI coordinate plane, and answer some questions concerning the plot:

1. What kind of relationship do you notice?
2. What is the rate of change?
3. What does the rate of change represent in the context of this problem?
4. Write the model for the voltage (V) in terms of current (I).

Figure 1. Sample Circuit to Illustrate Ohm's Law

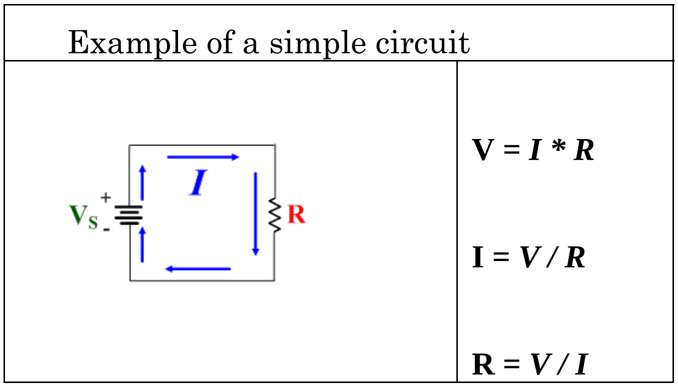


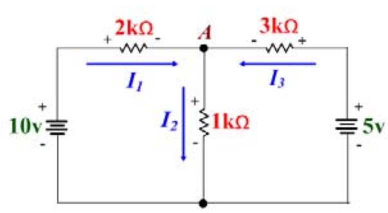
Table 2. List of Current Values Accross a Resistor and Corresponding Voltage Values

Current (I, in Amps)	Voltage (V, in Volts)
2	5
4	10
6	15
8	20
10	25

d. Systems of Linear Equations as Math Models for Simple Circuits via Kirchhoff's Laws

In this section, an example circuit containing several voltage sources and resistors is given (**Figure 2**).

Figure 2. Establishing a Linear System of Equations for a Simple Circuit



Then, the process of establishing a linear system of equation is presented in class by the teacher. For this particular example, Kirchhoff's Voltage Law is used to list two linear equations as follows:

$$\begin{cases} 10v - 2000\boldsymbol{I}_1 = 1000\boldsymbol{I}_2 \\ 5v - 3000\boldsymbol{I}_3 = 1000\boldsymbol{I}_2 \end{cases}$$

In addition, based on Kirchhoff's Current Law, $I_2 = I_1 + I_3$. Therefore, all three currents can be solved based on the three equations. Further, the voltage drop across any resistor can be obtained.

As an exercise, students are asked to solve the linear system of equations for I_1, I_2 and I_3.

e. Take Home Assignment

The assignment for students is to review all the materials in the teaching module, and solve another simple circuit problem by first establishing a linear system of equations.

Module 2: Using Trigonometric Identities to Analyze Signals in Wireless Receivers

This module introduces to students the signal processing that takes place in a simplified wireless communication receiver. To make our example understandable to high school students, we deliberately omitted several parts of practical wireless receivers, because the primary purpose of the teaching module is to show the usefulness of trigonometry in engineering applications.

a. Introduction

The introduction seeks to instill some fundamental knowledge of signal processing for modern communication systems. For example:

"A signal is a measurable varying phenomenon. It can be continuous (analog) or discrete (digital). For decades, audio and video signals were transmitted in analog form. With continued advancements in technology, digital signals are more often used in transmission and processing. We recently switched television signals to digital format.

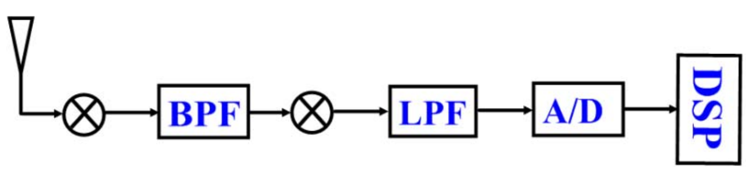
"Most communication systems use sinusoidal signals, such as your cell phone, television, radio, etc. At the receiver, we are only interested in one signal, but there are many other interfering signals that we don't want, yet still receive. Therefore, the receiver has to remove unwanted incoming signals. The design and analysis of wireless receivers involves heavy use of trigonometric identities and formulas.

"Why do we want a digital signal instead of an analog signal? Simply put, because digital signals only take values of 1's and 0's. When we receive digital signals corrupted by noise, it is still easy to determine whether a "1" or a "0" has been transmitted, as long as the noise is not too large. For example, if we receive a signal valued at 0.92, we know a "1" has been sent. There are always some noise or interference that makes the received signal not exactly 1. On the other hand, it is very difficult to remove the effect of noise for an analog signal. Also, computers only work with 1's and 0's, so digital signals are more compatible with computers."

b. A Simplified Receiver Diagram with Fundamental Components Introduced

The diagram (**Figure 3**) shows a simplified version of wireless receiver. Students need some explanation of the functionality of each block. The component symbols and their functionalities are summarized in **Table 3**.

Figure 3. A Wireless Receiver Diagram



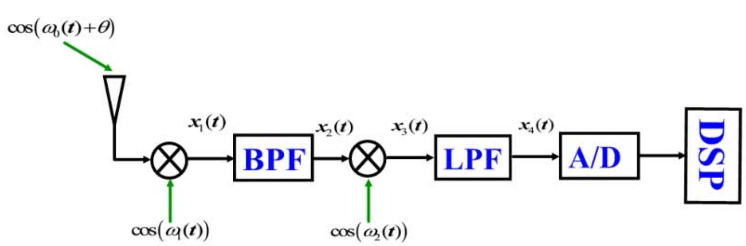
c. Signal Analysis at Each Stage

In order to perform signal analysis, signal symbols need to be defined and added to the receiver diagram (**Figure 4)**. Here, ω_0, ω_1, ω_2 are signals' frequencies that satisfy $\omega_0 = \omega_1 + \omega_2$; θ is determined by the information the transmitter is trying to send, e.g., $\theta = 0$ when the binary information bit is 1, and $\theta = \pi$ when the information bit is 0.

Table 3. Component Symbols and Functions in the Wireless Receiver

	Functions	Symbols
Antenna	Receives signals.	
Mixer	Multiplies two input signals and outputs the product.	$\otimes$
Band Pass Filter	Allows the desired components in a certain frequency band to pass and rejects the rest.	BPF
Low Pass Filter	Allows the desired low frequency components to pass and rejects the higher frequency components.	LPF
Analog to Digital Converter	Converts analog signals to digital ones.	A/D
Digital Signal Processor	Performs processing functions on digital signals.	DSP

Figure 4. Wireless Receiver Diagram with Signal Symbols Added



d. Signal Processing Analysis using Trigonometric Identities

The analysis process needs to be presented to the students by the math teacher in class. The purpose is to let students see the importance of trigonometry to the understanding of the received signal's transformation from radio frequency to baseband, before the transmitted information can be extracted.

1. Use Trigonometric Identity to find $x_1(t)$

$$\begin{aligned} x_1(t) &= \left[\cos\left(\omega_0(t)+\theta\right)\right]\cos\left(\omega_1(t)\right) \\ &= \frac{1}{2}\left[\cos\left(\omega_0(t)+\theta-\omega_1(t)\right)+\cos\left(\omega_0(t)+\theta+\omega_1(t)\right)\right] \\ &= \frac{1}{2}\left[\cos\left(\left(\omega_0-\omega_1\right)(t)+\theta\right)+\cos\left(\left(\omega_0+\omega_1\right)(t)+\theta\right)\right] \end{aligned}$$

The band pass filter will reject the unwanted frequency (the second term of $x_1(t)$) and allow the desired signal component, $x_2(t)$, to continue.

2. Find $x_2(t)$.

$$x_2(t) = \frac{1}{2}\cos\left(\left(\omega_0-\omega_1\right)(t)+\theta\right)$$

At the second mixer the signal $x_2(t)$ is multiplied by $cos(x_2(t))$.

3. Find $x_3(t)$.

$$\begin{aligned} x_3(t) &= \left[\frac{1}{2}\cos\left(\left(\omega_0-\omega_1\right)(t)+\theta\right)\right]\cos\left(\omega_2(t)\right) \\ &= \frac{1}{2}\left[\frac{1}{2}\left[\cos\left(\left(\omega_0-\omega_1\right)(t)+\theta-\omega_2(t)\right)+\cos\left(\left(\omega_0-\omega_1\right)(t)+\theta+\omega_2(t)\right)\right]\right] \\ &= \frac{1}{4}\left[\cos\left(\left(\omega_0-\omega_1\right)(t)+\theta-\omega_2(t)\right)+\cos\left(\left(\omega_0-\omega_1\right)(t)+\theta+\omega_2(t)\right)\right] \end{aligned}$$

4. Simplify $x_3(t)$.

$$\begin{aligned} x_3(t) &= \frac{1}{4}\left[\cos\left(\omega_2(t)+\theta-\omega_2(t)\right)+\cos\left(\omega_2(t)+\theta+\omega_2(t)\right)\right] \\ &= \frac{1}{4}\left[\cos\left(\omega_2(t)-\omega_2(t)+\theta\right)+\cos\left(\omega_2(t)+\omega_2(t)+\theta\right)\right] \\ &= \frac{1}{4}\left[\cos\left(\theta\right)+\cos\left(2\omega_2(t)+\theta\right)\right] \end{aligned}$$

The Low Pass Filter rejects any unwanted high frequency component (the second term of $x_3(t)$) and allows the desired low frequency signal, $x_4(t)$, to continue.

5. $x_4(t)$ only contains the baseband signal $cos(\theta)$, which can be used to extract the transmitted information bit (either 1 or 0). In practice, the baseband signal needs to go through the A/D converter and the DSP for further digital processing, before the information can be obtained and used (e.g., delivered to a speaker).

e. Take Home Assignment

As a homework assignment, students are required to perform the signal analysis in the wireless receiver, given specific values of θ, such as $\theta = 0, \pi/4, \pi/2, 3/4\ \pi,$ and π.

Comments and Conclusions

Our developed teaching modules are especially suitable for high school algebra and trigonometry courses. After the solution of linear systems of equations or trigonometry identities are covered, the teaching modules are presented to inspire students' interests in engineering and motivate them to master essential math skills. In order to promote the use of the teaching modules among math teachers, a teacher's edition (with solutions to exercises and assignments) and a student handout are developed separately for each module. The materials are flexible enough that teachers can adapt the teaching modules according to their students' needs. Also, we believe teachers should be first introduced to the materials through workshops or peer mentoring, so they will feel comfortable to present the materials and be able to answer possible questions raised by students. Collaboration between university faculty members and K-12 teachers is especially beneficial when preparing for the delivery of teaching modules.

At the end of July 2010, all ERAU RET teachers presented the teaching modules they developed over the six-week period. Our modules were well received by teachers and ERAU faculty members, especially because they were the only modules developed for math classes. It is particularly worth mentioning that, we received a very positive comment from an ERAU undergraduate student assistant, who worked for the RET program and attended the presentation. She indicated how she wishes she was exposed to such practical engineering examples in high school math classes, and how positive the effect could be on students' math study and interests in choosing engineering as their major in college.

Currently, Mr. Fernandez is incorporating the two teaching modules in his classes, and he is also seeking to disseminate the materials among math teachers in Miami-Dade County Public Schools. In the future, we plan to continue with our development of teaching modules for high school math courses, promote the use of these modules through publications and workshops, and collect assessment data as our proposed teaching practice is implemented for a large number of math classes.

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Jose Fernandez

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