Independent Research in Dynamics, a Case Study in Student Learning

Hodge E. Jenkins[1]

Abstract – While the typical undergraduate student who pursues research is generally a top performer academically, less-gifted students might benefit from a conducting a research investigation of their own interest. This paper chronicles an independent study and research of a non-research oriented, student in a single semester independent project. The student (average to slightly below average academically) chose his problem for study, with the approval and guidance of a supervising faculty. The academic goals of this effort were to achieve an increased level of understanding in applications of dynamic systems modeling to design, simulation, and ultimately to answer an engineering design question, relating to the dynamics of a real system.

A motorcycle rear suspension design was the specific topic of the investigation. The student, a motorcycle rider, used the Honda Shadow VT750 Aero for the basis of his investigation, as it is a mid-size motorcycle that utilizes a standard motorcycle suspension design. The suspension components studied by the student researcher consisted of the front fork, and the rear swing arm. A standard design motorcycle suspension has a spring and damper on both sides of the fork and rear swing arm. Both the rear and the front suspension lines of action are offset from the vertical by an angle. The initial questions investigated by the student were "How does the angle of orientation affect the rider comfort and handling?" and "Is there a general design optimum possibly based on rider comfort characteristics?".

Mainly through the process of developing an appropriate mathematical model, simulating road conditions (input), and examining simulation results the student had the opportunity achieve a higher level of knowledge and skill in dynamics and its application to engineering design, as well as develop critical thinking skills.

Although the results, conclusions, and recommendations revealed a less than a thorough investigation of the subject, they did demonstrate a more comprehensive grasp of modeling by the student. The student work is presented along with critiques and comments. Motivation by the student to perform the research seemed to be a significant factor in the final results.

Keywords: independent study, dynamics, undergraduate research.

Introduction

Different modes of learning vary from reading, listening, watching, and doing. By far, experiential learning (doing) has the most lasting impact on later performance of the same or similar task. Thus, it is no surprise that students may achieve significantly greater understanding and proficiency by being actively engaging in independent research.

There has been a significant number of studies to date, showing the potential benefits of undergraduate student research. As noted in a 1989 report from the National Science Foundation [1], "it is clear that the academic community regards the involvement of undergraduate students in meaningful research…with faculty members as one of the most powerful instructional tools." A more emphatic statement from the National Research Council is that "research is a necessary component of the bachelor's degree education [2]." Others have also documented specific programs that have had success in student learning and skills [3].

[1] Professional affiliation, mailing address, and e-mail address of first author

Those who question the validity of the impact and usefulness of undergraduate research point to content and cost of the effort. In an attempt to assess the impact of undergraduate research, the University of Delaware conducted extensive surveys of alumni (including those who did and did not perform undergraduate research) [4]. It was found that those who had undergraduate research experience reported enhanced important cognitive and personal skills associated scientific inquiry and critical thinking. Not surprisingly, these students were also more likely to attend graduate school.

The evidence of the aforementioned study highlights the educational enhancement of limited numbers of high achieving students. For example the University of Delaware study indicated that only 25 out of 1,000 engineering students participated in undergraduate research.

This paper details the independent study and research of an academically average to slightly below average student in a single semester project. The student had no plans to attend graduate school either before or after completing his research. The student selected his project with the approval and guidance of a supervising faculty. In the area of dynamic systems modeling, simulation, and design the student was tasked to assess the effects of the angle variation in vehicle suspension design and determine if there is an optimum.

The work of the student is presented along with a critique of the effort. While the observations of this manuscript of the particular student research are anecdotal, they do provide some insight into the achievable results of an average student, atypical of the most students involved in independent research.

PROBLEM DESCRIPTION

The student chose his specific problem for study, from a potential list provided by the supervising faculty member. Being an avid motorcyclist, the student decided to investigate suspension dynamic design issues for a motorcycle. Specifically, a Honda Shadow VT750 Aero, a mid-size motorcycle that utilizes a standard motorcycle suspension design, Figure 1, was chosen to be the basis of the study. The motorcycle suspension consisted of the front fork, and the rear swing arm. The motorcycle's front suspension has a spring and damper (shock absorber) on each side of the front fork as it extends from the front wheel axel to the handlebars. Both the rear and the front suspension are offset at an angle. Figure 1 shows a 2001 model of the Honda Shadow VT750 Aero motorcycle with its rear shocks angled at 45º from vertical and its front shocks at 34º from vertical.

The student was to develop a multi-degree of freedom variable model of the motorcycle, a road excitation input and perform simulations of the motorcycle with this input. A goal of the study was to determine the effect of rear suspension angle on ride performance. Ride performance was to be established by the student, using the simulation results for acceleration of a rider on the seat.

The student was presented with a previous study of motorcycle [4]. However, the previous study was only lightly examined by the student.

It should be noted that the student did not have a vibration course prior to this independent study, only an advanced dynamics course that contained an introduction to vibration.

Figure 1. Honda Shadow VT750 Motorcycle [R1]

Student Interaction and Independence

The independent study presented here was a single semester project by a graduating senior. The study was embodied as a 1-credit course for a letter grade. Student motivation for the research project was to satisfy a missing credit requirement for graduation. Two initial meetings with the supervising faculty were held with the student to select the research problem and to review expectations. Afterwards, the student met with the supervising faculty on an as needed basis, with usually no more than 2-weeks between meetings to discuss problems and progress. All work was performed in a single semester and documented in a final research paper.

The student performed his work by himself. During the student-faculty meetings details were reviewed. The student also required a short review (less than 30 minutes) of several vibration and dynamic system concepts at many of the initial meetings. The student had difficulties at nearly all stages of the work (modeling, assumptions, with software). The student had difficult trouble with his mathematical modeling as well as programming, and required some additional faculty time to correct such problems.

MODEL DEVELOPMENT

The student developed a three degree of freedom representation, limiting the model to vertical displacements. Figure 2 is the schematic of the model, where x_1, x_2, and x_3 represent the vertical displacements of the rear wheel, front wheel, and the central body of the motorcycle, respectively. All dynamic properties (mass, stiffness, damping) were all determined for the specific motorcycle from physical measurements by the student as part of the research. The student performed well at determining these parameters for the motorcycle, even visiting showrooms to see alternate designs, with no significant faculty effort.

The particular system outputs examined by the student were the displacement, velocity, and forces imposed on the seated rider for given a specific road profile. A road profile input was also part of the student's work.

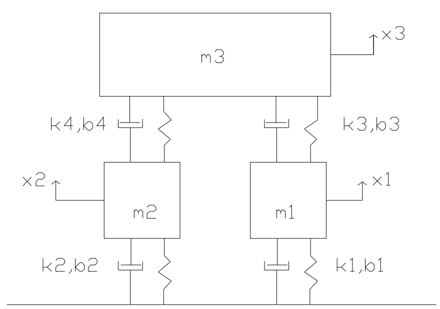

Figure 2. Three Degree of Freedom Model (Unidirectional)

The rear tire mass, stiffness and damping are represented by m_1, k_1, and b_1, while m_2, k_2, and b_2.represent the properties of the front tire. The rear suspension stiffness and damping are given by k_3 and b_3, and, and mass of the motorcycle central body is represented by m_3. Using plane motion constraints, a lumped parameter dynamic model was created using Newton's second law of motion (1), applied at each of the three lumped masses (or degrees of freedom). The student did fairly well working independently up to this point.

$$\sum \vec{F} = m\vec{a} \tag{1}$$

The road displacements at the front and rear tires were represented by, u_f, and u_r and were used as inputs to the model, with forces generated through the tire stiffness and damping. The free body diagrams of each mass were used to develop the three dynamic equations.

$$m_1\ddot{x}_1 = b_3\dot{x}_3 + k_3x_3 - (b_3 + b_1)\dot{x}_1 - (k_3 + k_1)x_1 + \dot{u}_f b_1 + u_f k_1 \tag{2}$$

$$m_2\ddot{x}_2 = b_4\dot{x}_3 + k_4x_3 - (b_2 + b_4)\dot{x}_2 - (k_2 + k_4)x_2 + \dot{u}_r b_2 + u_r k_2 \tag{3}$$

$$m_3\ddot{x}_3 = b_4\dot{x}_2 + k_4x_2 - (b_3 + b_4)\dot{x}_3 - (k_3 + k_4)x_3 + b_3\dot{x}_1 + k_3x_1 \tag{4}$$

The student had only slight difficulty in the development of the dynamic equations. A bit of a review from the supervising faculty on the equation of motion for a simple 1-DOF system was necessary to prime the student to create the complete 3-DOF model. Also the student had difficulties in detecting small errors in his equations.

State Space Representation

After completing the mathematical model of the system, correctly, the student had the choice of modeling the system using either Simulink™ or Matlab™ software. The Simulink™ program provides a very graphical user environment with function blocks connecting inputs and outputs compared to the text/script format of Matlab™. Figure 3 depicts a typical model of a 2-DOF system (not what the student used). It is interesting to note that most students without prior experience in either of the two software packages, prefer Simulink™ because of its graphical, drag-drop-connect nature, for 1-DOF (single mass) systems [7]. However as models increase in the number degrees of freedom, they become visually more complicated, with numerous connections between function blocks. The student involved in this motorcycle study decided it was more effective to use Matlab™, with a state space representation of his 3-DOF system, after an initial unsuccessful trial with a Simulink™ model. The Matlab™ script model of student's system (Fig. 2) is provided in the Appendix. All work presented by the student was performed using Matlab™.

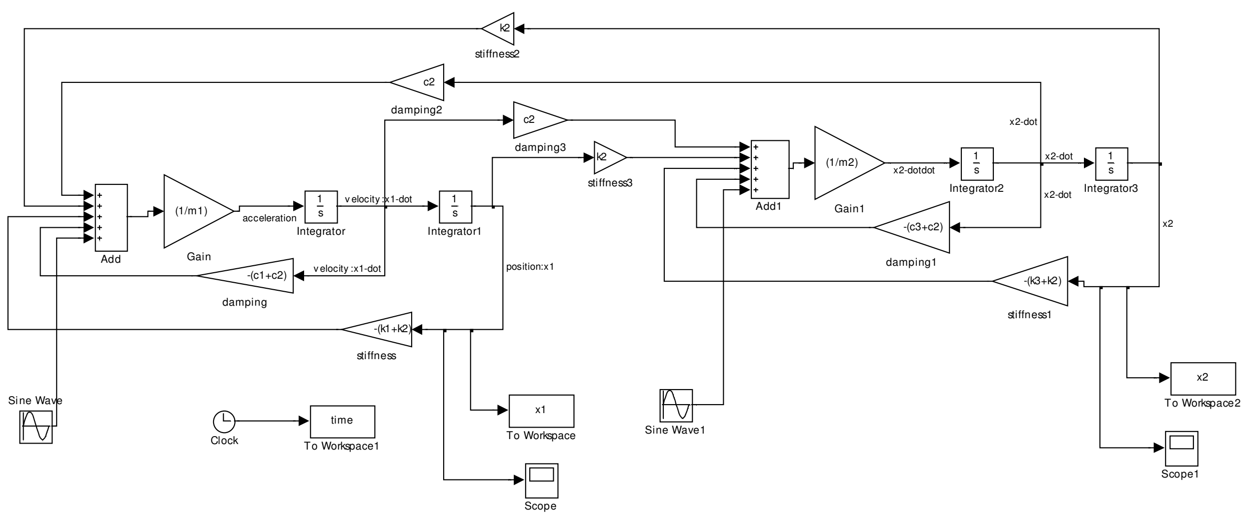

Figure 3. Simulink Model for a Two Degree of Freedom System

The use of state space forms of the equations of motion are most easily handled in Matlab™ for this lumped parameter, linear time invariant (LTI) system. Six, first order, linear differential equations were created, from the three 2nd order linear differential equations (2), (3), and (4). The general form of state space equations can be represented as

$$\{\dot{q}(t)\} = [A]\{q(t)\} + [B]\{u(t)\} \tag{5.a}$$

$$\{y(t)\} = [C]\{q(t)\} + [D]\{u(t)\} \tag{5.b}$$

With *{y(t)}* being the output vector and *{u(t)}* is the input vector (road shape as a function of time for the front and rear wheels). The states, *{q(t)}* are defined by Eq. (6) to be the positions and velocities of the three masses.

$$\{q(t)\} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} \tag{6}$$

The seven outputs, *{y(t)}* were defined by Eq. (7). These are the six states and the net force on the main body.

$$\{y(t)\} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ force = b_4\dot{q}_2 + k_4 q_2 - (b_3 + b_4)\dot{q}_3 - (k_3 + k_4)q_3 + b_3\dot{q}_1 + k_3 q_1 \end{bmatrix} \tag{7}$$

The input vector included the front and rear road displacements and velocities and the forces generated through the springs and masses on the body

$$\{u(t)\} = \begin{bmatrix} u_f \\ u_r \end{bmatrix} \tag{8}$$

The state space equations (5.a) and (5.b.) resulted in the following systems of equations.

$$\dot{q}_1 = q_4 \tag{9}$$

$$\dot{q}_2 = q_5 \tag{10}$$

$$\dot{q}_3 = q_6 \tag{11}$$

$$\dot{q}_4 = (1/m_1)[b_3 q_6 + k_3 q_3 - (b_3 + b_1)q_4 - (k_3 + k_1)q_1 + \dot{u}_f b_1 + u_f k_1] \tag{12}$$

$$\dot{q}_5 = (1/m_2)[b_4 q_6 + k_4 q_3 - (b_2 + b_4)q_5 - (k_2 + k_4)q_2 + \dot{u}_r b_2 + u_r k_2] \tag{13}$$

$$\dot{q}_6 = (1/m_3)[b_4 q_5 + k_4 q_2 - (b_3 + b_4)q_4 - (k_3 + k_4)q_3 + b_3 q_4 + k_3 q_1] \tag{14}$$

Equations 9-14 provide the state equations with 6 state variables that can be solved using the state space method in Matlab^{TM} . The matrices in the state equations represent the coefficients in equations 9-14, where ***A*** is the state matrix, ***B*** is the input matrix, ***C*** is the output matrix and **D** is the feedthrough matrix. Quantitative values of these matrices can be found in Appendix. Again the student had problems making his model work properly, lacking sufficient troubleshooting skills. The supervising faculty reviewed and corrected the model. The student had made a few typing errors in the model.

Model Validation

To assess the model's validity the student examined the model's natural frequencies. Using the model defined, (eqns. 9-14) the eigenvalues of the model were determined using equation (15),

$$\lambda[m]\{\ddot{x}\} + [k]\{x\} = 0 \quad (15)$$

where λ is the eigenvalue, [m] is the mass matrix, and [k] is the stiffness matrix. The natural frequencies of the system can then be represented as

$$f_i = \frac{\omega_i}{2\pi} = \frac{\sqrt{\lambda_i}}{2\pi} \quad i=1, 2, 3 \quad (16)$$

where f_i is natural frequency in cycles per second (Hz), ω_i is natural frequency in radians per second (square root of the eigenvalue). The first natural frequency from the state space model was compared to a simplified equivalent stiffness model shown in Figure 4. The student was able to create a 1-DOF model based on the 3-DOF model.

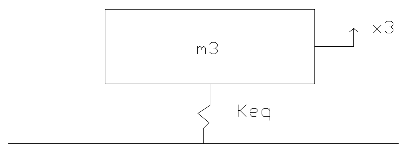

Figure 4: Simplified motorcycle using equivalent stiffness

Model Input

A road profile was developed for input that consisted of two sinusoidal functions (Eq. 17) which was applied as input to both the front and rear wheels, simultaneously (phase differences were not included). The student noted a realistic road profile would be completely random with a significant range of frequency content, but did not include that in his model input.

$$u_f = u_r = \sin(t) + \sin(2t) \text{ [inches]} \quad (17)$$

The student had difficulty in development of the input for the model. Clearly the input road profile could be a function of many variables, including speed. Thus, a poor compromise of a two function input was reached by the student. This difficulty is not uncommon for students exploring open-ended vehicle suspension problems. It is interesting to note that the student recognized this limitation of his created input, yet did not act on it. No investigation of previous work for development of a road input model was put forth by the student.

Simulations and Results

Using the model road input of eq. 17, several different combinations of spring rates were simulated to compare the vertical displacement, velocity, and forced imparted to the motorcycle body. Table 1 provides the results of the motorcycle response (displacement, velocity, force) to different front spring rates, while holding the rear spring rate constant. Similarly, Table 2 shows the motorcycle response to different rear spring rates, while holding the front spring rate constant. A graphical representation of a typical simulation can be seen in Figure 5. Note: the formats of Tables 1 and 2 are in the student's original presentation format.

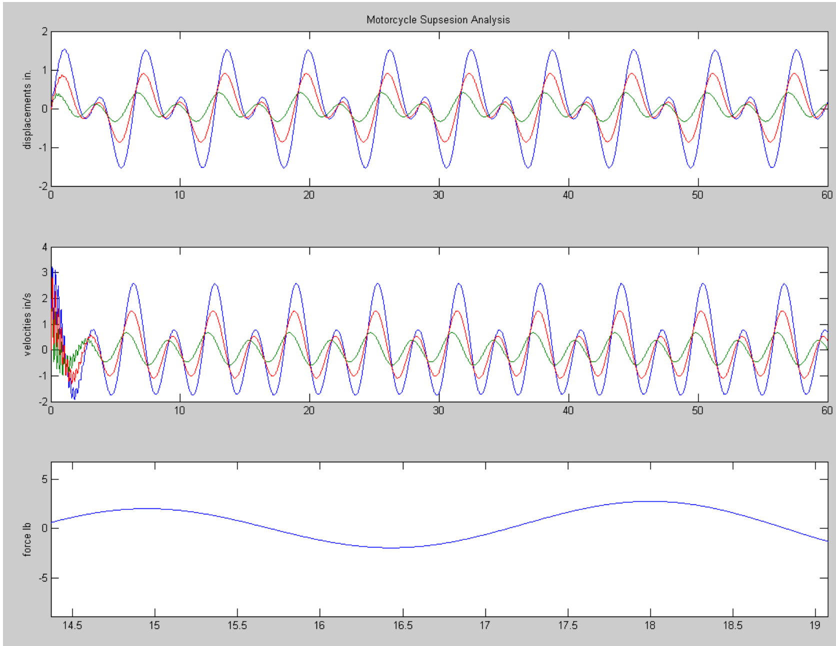

Figure 5: Displacement, Velocity, and Force Typical simulation output.

Table 1: Student Results for Maximum displacement, velocity, and body forces using different front spring rates holding the rear spring at 120 lb/in

Front Spring rate[lb/in]	Max Displacement [in]	Max Velocity [in/s]	Max Force [lb]
50	0.9224	2.805	44.655
60	0.9137	2.805	44.66
70	0.9063	2.806	44.665
80	0.9	2.807	4.67
90	0.8948	2.808	44.675

Table 2: : Student Results Maximum displacement, velocity, and body forces using different rear spring rates holding the front spring at 80 lb/in

Rear Spring rate[lb/in]	Max Displacement [in]	Max Velocity [in/s]	Max Force [lb]
100	0.8936	2.8074	44.672
110	0.8971	2.8072	44.671
120	0.9	2.807	4.67
130	0.9024	2.8068	44.669
140	0.9044	2.8066	44.669

DISCUSSION AND CONCLUSIONS

Discussion and conclusions from the student were limited. The student stated that the results from the simulations studies indicated that a lower spring rate in the rear and a higher spring rate in the front would decrease the displacement of the rider. The 1st natural frequency of the 3-DOF model was 1.917 [Hz]. The natural frequency of the using the equivalent spring , 1-DOF model was 1.969 [Hz].

Additional concluding remarks from the student had no foundation upon his results, but were interesting comments relating to motorcycle suspension design, not previously discussed in background. No observational comments relating to very small range of variation in the results, regardless of the spring rate differences were mentioned by the student.

SUMMARY CRITIQUE

The student who performed this research made a great deal of progress in learning system dynamics and creating multi-degree of freedom models of a real physical system. Skills were gained in using professional software (Matlab™) by the student. However, the desired outcome of enhanced critical thinking may not have been achieved. While the student's results seem to show that the spring rate was inconsequential, the student indicated a preference based on negligible differences in displacement, and a comment about frequency results. Displacement maximums have little bearing on the rider comfort. The student would have gained more from examining the net vertical force on the car body.

The discussion of the results by the student was very limited and did not reflect on the actual results but only a general discussion of the parameters studied in terms of current design philosophies. This was disappointing. The student had come very far in modeling, application of dynamic equations, simulation, and software knowledge. However, the synthesis of new knowledge had escaped him. This may be partly because of the limited amount of analysis, effort, and lack of curiosity or interest in the final results. Note: the student was a graduating in the semester the study was conducted. As previously mentioned, the student was in the lower half of the class, academically.

Further disappointment in the student work can be seen in the total absence of an explanation of why all the simulation results appeared to be about the same, regardless of the variation in shock stiffness. (I believe this may be due to the overriding low stiffness of the tires in the model.) The student may have run out of available time or had no troubleshooting knowledge. Documentation and note keeping skills of the student were also lacking.

Lastly the student did not tie the analysis and results to the original design question of "How does the angle of orientation affect the rider comfort and handling?". This was never answered.

Many shortcomings of the student effort may be due the real lack of motivation of the student to do the independent study. The student described in this paper, was not motivated to perform research. Motivation was completion of a necessary degree requirement. Although the student had a strong interest in motorcycles, he lacked the appropriate academic curiosity to perform well in research.

In summary, this student's performance shows positive and negative aspects of less-gifted students performing research. Gains in knowledge and understanding were achieved, but investigative focus and critical thinking were not observably enhanced.

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Hodge Jenkins

Dr. Hodge Jenkins is an Associate Professor in the Department of Mechanical Engineering at Mercer University in Macon, Georgia. Prior to coming to Mercer in 2002, Dr. Jenkins was employed in optical fiber product development with Bell Laboratories of Lucent Technologies. He is a registered professional engineer, and with over 20 years of design and development experience in high-precision machine design, dynamic analysis, process automation, control, and robotics. Dr. Jenkins holds a Ph.D. in Mechanical Engineering from Georgia Institute of Technology in (1996), as well as BSME (1981) and MSME (1985) degrees from the University of Pittsburgh. His professional affiliations include ASEE and ASME.

Appendix Matlab Script for Motorcycle

```
%% Motorcycle suspension
%%
g=386.4; %%in^2/s

m1=35/g; %%lb/g
m2=35/g; %%lb/g
m3=450/g; %%lb/g

k1=500; %%lb/in
k2=500; %%lb/in
k3=80; %%lb/in
k4=140; %%lb/in

c1=0.0036; %%lb*s/in
c2=0.0036; %%lb*s/in
c3=170; %%lb*s/in
c4=170; %%ls*s/in


A=[0 0 0 1 0 0
0 0 0 0 1 0
0 0 0 0 0 1
-(k1+k3)/m1 0 k3/m1 -(c1+c3)/m1 0
c3/m1
0 -(k2+k4)/m2 k4/m2 0 -(c2+c4)/m2
c4/m2
k3/m3 k4/m3 -(k3+k4)/m3 c3/m3 c4/m3
-(c3+c4)/m3];

B=[0 0 0 0
 0 0 0 0
 0 0 0 0
 k1/m1 c1/m1 0  0
 0 0 k2/m2 c2/m2
 0 0 0 0];

C=[1 0 0 0 0 0
   0 1 0 0 0 0
   0 0 1 0 0 0
   0 0 0 1 0 0
   0 0 0 0 1 0
   0 0 0 0 0 1
   k3 k4 -(k3+k4) c3 c4 -(c3+c4)];
%%Y=CX +DU
Y=[x1,x2,x3,x1dot,x2dot,x3dot,
force onseat]
```

```
D=[0 0 0 0
   0 0 0 0
   0 0 0 0
   0 0 0 0
   0 0 0 0
   0 0 0 0
   0 0 0 0];

t=(0:0.005:60);

F1=(sin(1*t)+sin(2*t))';
F2=F1;
Ft=[F1 F2 zeros(length(t),2)];

[Y,X]=lsim(A,B,C,D,Ft,t);

max(Y)
min(Y)

M=[m1 0 0
   0 m2 0
   0 0 m3]

K=[-(k1+k3) 0 k4
    0 -(k2+k4) k4
    k3 k4 -(k3+k4)]

Z=inv(M)*-K;

eigen=eig(Z)
natfeq=sqrt(eigen)/(2*pi)

Keq=inv(inv(k3)+inv(k1))+inv(inv(k2
)+inv(k4))
eigenEQ=Keq/m3
natfeqeq=sqrt(eigenEQ)/(2*pi)

subplot(3,1,1), plot(t',Y(:,1:3)),
ylabel('displacements in.'),
title('Motorcycle Suspension
Analysis');;
subplot(3,1,2), plot(t',Y(:,4:6)),
ylabel('velocities in/s');
subplot(3,1,3), plot(t',Y(:,7)),
ylabel('force lb');
```